

1.2 Integration by Substitution

Exercises

1. Find the following integrals:

(a) $\int 9(x^2 + 3x + 5)^8(2x + 3) dx$;

(b) $\int x^3 e^{x^4+2} dx$;

(c) $\int e^{6x-1} dx$;

(d) $\int e^{1-x} dx$;

(e) $\int \frac{3x}{x^2-1} dx$;

(f) $\int \frac{3x+6}{\sqrt{2x^2+8x+3}} dx$;

(g) $\int \frac{(\ln x)^2}{x} dx$;

(h) $\int \frac{x}{x+1} dx$;

(i) $\int \sqrt{4x+1} dx$;

(j) $\int \frac{1}{7x+6} dx$;

(k) $\int \frac{1}{x(\ln x)^2} dx$;

(l) $\int \frac{\ln x^2}{x} dx$;

(m) $\int \frac{2x \ln(x^2+1)}{x^2+1} dx$;

(n) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$;

2. The resale value of a certain industrial machine decreases at a rate that changes with time. When the machine is t years old, the rate at which its value is changing is $-960e^{-t/5}$ euros per year. If the machine was bought new for 5,000 €, how much will it be worth 10 years later?

3. It is estimated that t years from now, the value $V(t)$ of an acre of farmland will be increasing at the rate of

$$V'(t) = \frac{0.4t^3}{\sqrt{0.2t^4 + 8,000}}$$

euros per year. The land is currently worth 500 € per acre.

- (a) Find $V(t)$.
 - (b) How much will the land be worth in 10 years.
 - (c) When will the land be worth 1,000 € per acre?
4. MacCell corporation has set up a production line to manufacture a new type of cellular telephone. The rate of production of telephones is

$$\frac{dx}{dt} = 1500 \left(2 - \frac{t}{2t+5} \right)$$

units per month. How many telephones are produced during the third month? **Hint:** Find $x(3) - x(2)$.

5. At a certain factory, the marginal cost is $3(q - 4)^2$ euros per unit when the level of output is x units.
- (a) Express the total production cost in terms of the overhead (fixed cost – the cost of producing 0 units) and the number of units produced.
 - (b) What is the total cost of producing 14 units if the overhead is 436 €?
6. In a certain section of the country, the price of chicken is currently 3 € per kilogram. It is estimated that t weeks from now the price will be increasing at the rate of $3\sqrt{t+1}$ cents per week. How much will chicken cost 8 weeks from now?
7. The price p (euros) of each pair of Xike sports sneakers is estimated to be changing at the rate

$$p'(x) = -\frac{150x}{(144 + x^2)^{3/2}},$$

where x (hundred) units is the consumer demand. Suppose 500 pairs of sneakers ($x = 5$) are demanded when the price is 75 € per pair.

- (a) Find the price function $p(x)$ with respect to demand.
- (b) At what price will 400 pairs of sneakers be demanded? At what price will no pairs be demanded?

- (c) How many pairs will be demanded at a price of 60 € per pair?
 - (d) Find the revenue function and the marginal revenue function. For what quantity x demanded is revenue maximized?
8. The owner of a fast food restaurant chain estimates that the price in euros of her newest product, Burger Babies, is changing at the rate

$$p'(y) = \frac{30y}{(3 + y^2)},$$

when y (thousand) burgers are supplied for purchase. The initial price is 2.25 € per burger.

- (a) Find the price function $p(y)$ with respect to supply.
 - (b) At what price will 4,000 additional burgers ($y = 4$) be supplied?
 - (c) How many more burgers will be supplied at a price of 3 € per burger?
9. A company determines that the marginal revenue from the production of x units is $7 - 3x - 4x^2$ hundred euros per unit, and the corresponding marginal cost is $5 + 2x$ hundred euros per unit. By how much does the profit change when the level of production is raised from 5 to 9 units?
10. A company determines that the marginal revenue from the production of x units is $\frac{11-x}{\sqrt{14-x}}$ hundred euros per unit, and the corresponding marginal cost is $2 + x + x^2$ hundred euros per unit. By how much does the profit change when the level of production is raised from 5 to 9 units?