Chapter 1

Differentiation

1.1 The Derivative: Slope and Rates

Exercises

1. For the following functions, calculate the derivative and find the slope of the tangent line to the graph for the given value of the independent variable:

   (a) $f(x) = 5x - 3$, $x = 2$;
   (b) $f(x) = x^2 - 1$, $x = -1$;
   (c) $f(x) = 2x^2 - 3x - 5$, $x = 0$;
   (d) $f(x) = x^3$, $x = -1$;
   (e) $f(x) = x^3 - 1$, $x = 2$;
   (f) $g(t) = \frac{2}{t}$, $t = \frac{1}{2}$;
   (g) $f(x) = \frac{1}{x^2}$, $x = 2$;
   (h) $f(x) = \sqrt{x}$, $x = 4$;
   (i) $h(u) = \frac{1}{\sqrt{u}}$, $u = 9$.

2. For the following functions, calculate the derivative and find the equation of the tangent line to the graph for the given value of x_0:

(a) $f(x) = x^2 + x + 1$, $x_0 = 2$;

(b) $f(x) = x^3 - x$, $x_0 = -2$;

(c) $f(x) = \frac{3}{x^2}$, $x_0 = \frac{1}{2}$;

(d) $f(x) = 2\sqrt{x}$, $x_0 = 4$.

3. For the following functions, find the rate of change $\frac{dy}{dx}$ when $x = x_0$:

   (a) $y = 3$, $x_0 = 2$;

   (b) $y = 6 - 2x$, $x_0 = 3$;

   (c) $y = x(1 - x)$, $x_0 = -1$.

4. For the following functions, draw the graph, then determine the values of x for which the derivative is zero. What happens to the graph at the corresponding points?

   (a) $f(x) = x^2 - 2x$;

   (b) $f(x) = x^3 + 3x^2$;

   (c) $f(x) = x^3 - x^2$.

5. A manufacturer cam produce a certain commodity at a cost of 20 € per unit. It is estimated that if the product is sold at a price of p euros per unit, the consumers will bye $120 - p$ units every month. Find the manufacturer's monthly profit $P(p) = R(p) - C(p)$, where $R(p)$ is the revenue and $C(p)$ is the cost. Is the profit increasing or decreasing when the price of the product is 60 € apiece? What about the price of 80 € apiece? What happens to the profit when the commodity is sold for 70 € per unit?

6. If the demand for a product is given by

$$D(p) = \frac{1{,}000}{\sqrt{p}} - 1$$

   what is the average rate of change of demand when p increases from

   (a) 1 to 25;

(b) 25 to 100?

7. Suppose that the demand for a certain product is given by $x = 10{,}000 - 100p$, where p euros is the price per unit. What is the instantanious rate of change of consumer expenditure with respect to price at

   (a) any price p;

   (b) $p = 5$;

   (c) $p = 20$?