

1.2 Matrices and Determinants. Solving Systems of Three Equations by Cramer's Rule

Exercises

1. For matrices A , B , C , D given by

$$A = \begin{bmatrix} 1 & -4 & -1 \\ -2 & 0 & 3 \\ 7 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 3 & 1 \\ -2 & 1 & 1 \\ 6 & -1 & 4 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & -3 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$$

determine if the following operations are possible, and (if they are) find the results.

- (a) $A + B$;
 - (b) $A + C$;
 - (c) $A - B$;
 - (d) $B + D$;
 - (e) $C + D$;
 - (f) $B + A$;
 - (g) $B - A$;
 - (h) $A - D$.
2. Formulate the Cramer's rule for systems of two linear equations (in two variables).
3. By using Cramer's rule solve the following systems of linear equations.
- (a)
$$\begin{aligned} x + 2y &= 5 \\ 3x - 2y &= -1 \end{aligned}$$

$$\begin{array}{l} \text{(b)} \quad 3x + 2y = 19 \\ \quad \quad 3x - 2y = 11 \end{array}$$

$$\begin{array}{l} \text{(c)} \quad 12x - 7y = 110 \\ \quad \quad 8x - 3y = 90 \end{array}$$

4. Use Cramer's rule to solve the following systems of linear equations.

$$\begin{array}{l} \text{(a)} \quad 2x_1 + 3x_2 + x_3 = 9 \\ \quad \quad x_1 - 2x_2 + 3x_3 = 6 \\ \quad \quad 3x_1 + x_2 + 2x_3 = 8 \end{array}$$

$$\begin{array}{l} \text{(b)} \quad 2v_1 - 5 - v_2 + 3v_3 = 0 \\ \quad \quad -2v_3 - 3v_2 - 4v_1 = 8 \\ \quad \quad v_2 + 3v_1 - 4 - v_3 = 0 \end{array}$$

5. Nan's Nursery sells bushes, trees, and perennial flowers. Each bush costs 20 €, each tree costs 40 €, and each flower costs 2 €. To plant these bushes, trees, and flowers, Nan's charges 10 € per bush, 10 € per tree, and 1 € per flower. Also, Nan will insure the items she plants for 1 year at a charge of 4 € per bush, 6 € per tree, and 0.20 € per flower. A new homeowner has budgeted 1,000 € for purchases, 400 € for planting the purchases, and 150 € for insuring purchases from Nan's. How many bushes, trees, and flowers should the homeowner buy to exactly use up the money budgeted?
6. The Hiker and Biker Outfitting Shop makes packages of snacks to order. One customer likes peanuts, raisins, and chocolate chips but wants only two ingredients per snack. In peanut-raisin mixes she likes twice as many peanuts as raisins, in peanut-chocolate chip mixes she likes twice as many chocolate chips as peanuts, and in raisin-chocolate chip mixes she likes equal amounts of the two foods. She buys 10 grams of peanuts, 15 grams of raisins, and 30 grams of chocolate chips. Find the amounts of the various mixes which will meet her conditions and completely use up the foods she has purchases.

7. Ted's Toys makes toy airplanes, boats, and cars. The materials used are plastic, wood strips, and steel. Each airplane uses 100 grams of plastic, 25 centimeters of wood strips, and 200 grams of steel. Each boat uses 50 grams of plastic, 250 centimeters of wood strips, and 50 grams of steel; and each car uses 50 grams of plastic and 150 grams of steel. If Ted's has on hand 10,500 grams of plastic, 3,750 centimeters of wood strips, and 25,500 grams of steel, how many planes, boats, and cars should be made to use up all these supplies?
8. Rachael, Stephanie, and Tina are investing for retirement, using stocks, bonds, and money market funds. They use the following guidelines: Rachael wants half of her money in stocks and the rest split equally in bonds and money market funds. Stephanie wants her money split equally among all three areas, and Tina wants her money split equally between stocks and bonds. If the annual return on stocks is 9%, on bonds 6%, and on money market funds 3%, what is the total that each woman should invest in order that each of them gain 10,000 € from her investment?