Functions and Their Graphs

Functions

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Aims and Objectives

- Introducing the notion of a function
- Introducing the notion of composition of functions
- Applying functions into practical applications

Contents

Functions

Function

A *function* is a rule that assigns to each element in a set A exactly one element in a set B.
The set A is called the *domain* of the function
and the set of assigned elements in B is called the *range*.

Interpretations of a Function f

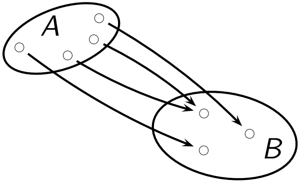

Figure: A function as a mapping

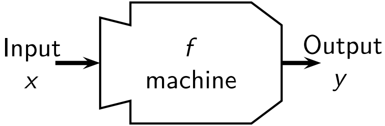

Figure: A function as a machine

Examples of Functions

Example

Find $f(2)$ if $f(x) = x^2 + 1$.

Solution.

$$f(2) = 2^2 + 1 = 5.$$

$\square$

- Functional relation: $y = f(x)$
- y – dependent variable
- x – independent variable

Examples of Functions. (Continued)

Example

Suppose the total cost in euros of manufacturing q units of a certain commodity is given by the function

$$C(q) = q^3 - 20q^2 + 600q + 300.$$

1. Compute the cost of manufacturing 10 units of the commodity.
2. Compute the cost of manufacturing the 10th unit of the commodity.

Examples of Functions. (Continued)

Solution.

1. The cost of manufacturing 10 units is the value of the *total cost function* when $q = 10$:

$$C(10) = 10^3 - 20 \cdot 10^2 + 600 \cdot 10 + 300 = 5300$$

euros.

2. The cost of manufacturing the 10th unit is the difference:

$$\begin{aligned} C(10) - C(9) = C(10) - (9^3 - 20 \cdot 9^2 + 600 \cdot 9 + 300) \\ = 5300 - 4809 = 491 \end{aligned}$$

euros.

$\square$

Composition of Functions

Composition of Functions

Given functions $g(u)$ and $f(x)$, the *composition* $g(f(x))$ is the function of x formed by substituting $u = f(x)$ for u in the formula for $g(u)$.

Interpretations of the Composition $g(f(x))$

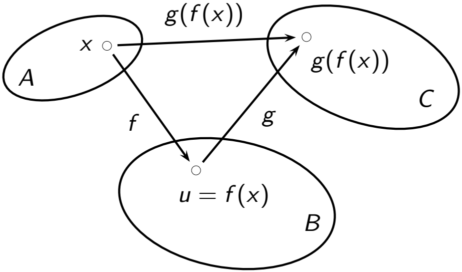

Figure: The composition as a mapping

Interpretations of the Composition $g(f(x))$. (Continued)

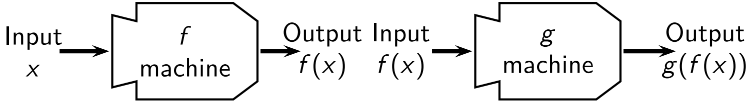

Figure: The composition as an assembly line

Examples of Compositions

Example

Find the composite function $g(f(x))$ if $g(u) = u^2 + 2u - 1$ and $f(x) = x - 1$.

Solution.

Replace u by $x - 1$ in the formula for $g(u)$ to get

$$\begin{aligned} g(f(x)) = g(x-1) &= (x-1)^2 + 2(x-1) - 1 \\ &= (x^2 - 2x + 1) + (2x - 2) - 1 = x^2 - 2. \end{aligned}$$

$\square$

Examples of Compositions. (Continued)

Example

Find $f(x+1)$ if $f(x) = 2x^2 - \frac{1}{x} + 1$.

Solution.

Write the formula for f in more neutral terms, e.g.:

$$f(\square) = 2(\square)^2 - \frac{1}{\square} + 1.$$

To find $f(x+1)$, insert the expression $x+1$ inside each box:

$$f(x+1) = 2(x+1)^2 - \frac{1}{x+1} + 1.$$

$\square$

For Further Reading

- `http://fberisha.netfirms.com`
- Homework: Exercises from teaching materials
- L. D. Hofmann, G. L. Bradley, *Calculus – for business, economics and life sciences,* pp. 2–16.
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks,* pp. 105–112.

Summary

- Function: dependency between two quantities
- Functional notation: $f(x)$
- Independent and dependent variables
- Composition of functions: $g(f(x))$