Linear Functions

F. M. Berisha

UNIVERSITETI I EJL
ЈИЕ УНИВЕРЗИТЕТ
SEE UNIVERSITY

South East European University, Tetovo

Aims and Objectives

- The notion of a linear function and sketching its graph.
- Constant rate of change. The slope of the line.
- Using the slope-intercept and point-slope equations of a line to solve application problems.

Contents

1. The Notion of Linear Function
   - A Linear Cost Function
   - The Notion of Linear Function and Its Graph
2. The Slope of a Line
   - The Notion of Slope
   - An Example
   - The Direction and Steepness of a Line
3. Equation of a Line
   - The Slope and y-Intercept
   - The Slope-Intercept Form
   - The Point-Slope Form

Application of a Linear Function

Example

A manufacturer's total cost consists of a fixed overhead of 100 € plus production costs of 60 € per unit.
Express the total cost as a function of the number of units produced and draw the graph.

Application of a Linear Function. (Continued)

Solution.

Denote by x the number of units produced
and by $C(x)$ the corresponding total cost.

$$[\text{Total cost}] = [\text{Cost per unit}] \cdot [\text{Number of units}] + [\text{Fixed cost}],$$

where are

$$\begin{aligned} [\text{Cost per unit}] &= 60, \\ [\text{Number of units}] &= x, \\ [\text{Fixed cost}] &= 100. \end{aligned}$$

Hence,

$$C(x) = 60x + 100.$$

The graph of the cost function is sketched in the following figure. □

The Graph of the Cost Function

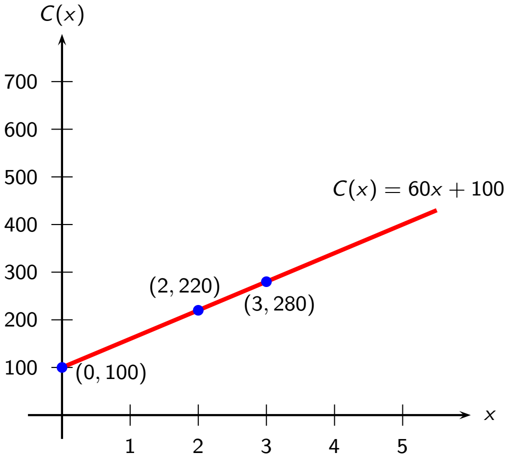

Figure: The cost function $C(x) = 60x + 100$.

The Notion of Linear Function

Linear Function

- A *linear function* is a function that changes at a constant rate with respect to its independent variable.
- The graph of a linear function is a straight line.
- The equation of a linear function can be written in the form

$$y = mx + b,$$

where m and b are constants.

The Slope of a Line

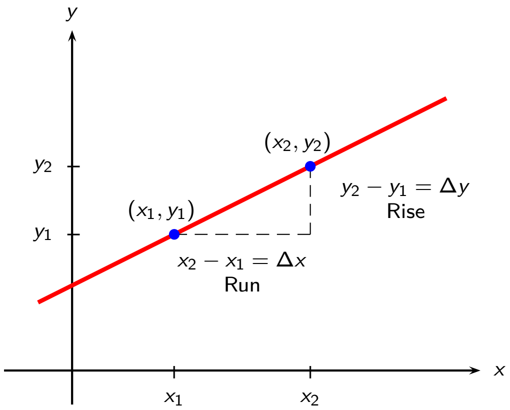

Figure: The slope $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$.

The Slope of a Line. (Continued)

- The slope is the ratio

$$[\text{Slope}] = \frac{[\text{Rise}]}{[\text{Run}]} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- Put $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$.

The Slope of a Line

The *slope* of a line passing through the points (x_1, y_1) and (x_2, y_2) is given by the formula

$$[\text{Slope}] = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example of Slope

Example

Find the slope of the line joining the points $(-3, 4)$ and $(2, -1)$.

Solution.

$$[\text{Slope}] = \frac{\Delta y}{\Delta x} = \frac{-1-4}{2-(-3)} = \frac{-5}{5} = -1.$$

The graph is drawn in the following figure. □

Example of Slope. (Continued)

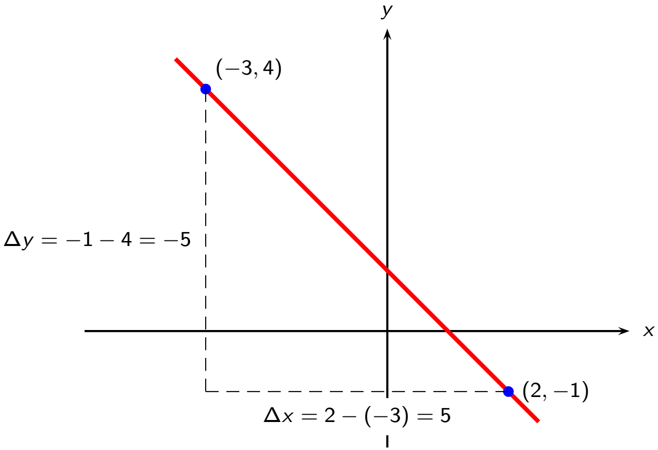

Figure: The line joining $(-3, 4)$ and $(2, -1)$.

The Direction and Steepness of a Line

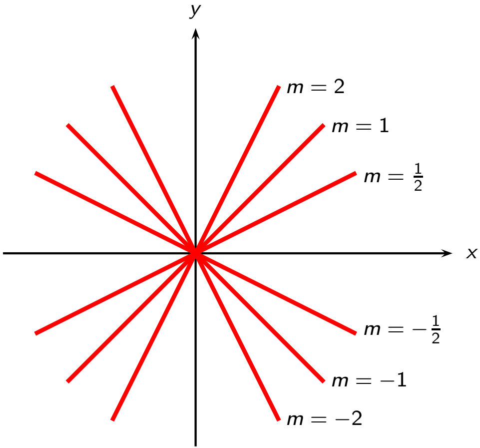

Figure: The direction and steepness of a line.

Horizontal and Vertical Lines

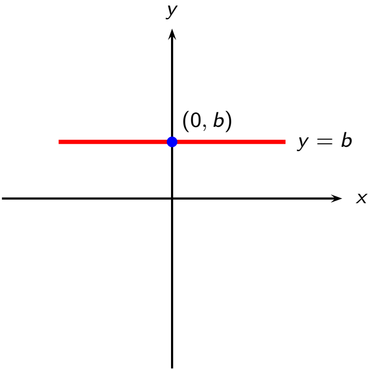

Figure: A horizontal line $y = b$.

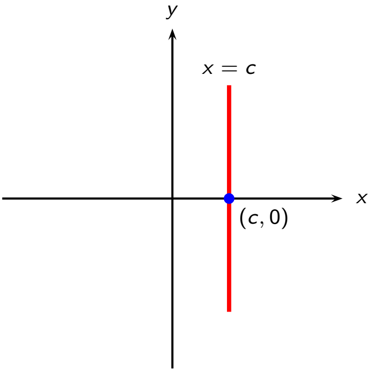

Figure: A vertical line $x = c$.

The Slope and y-Intercept

- The equation of a line:

$$y = mx + b$$

- The slope of the line:

$$\begin{aligned}[\text{Slope}] = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1} \\ = \frac{mx_2 - mx_1}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m\end{aligned}$$

- y-intercept of the line: $(0, b)$

The Slope and y-Intercept. (Continued)

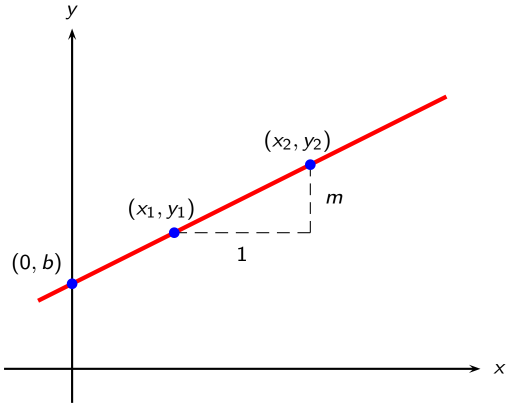

Figure: The slope and y-intercept of the line $y = mx + b$.

The Slope-Intercept Form of the Equation of a Line

The Slope-Intercept Form of the Equation of a Line

The equation

$$y = mx + b$$

is the equation of the line whose slope is m
and y-intercept is $(0, b)$.

An Example of the Slope-Intercept Form

Example

Find the slope and y-intercept of the line $3y + 2x = 6$.

Solution.

Rewrite the equation in slope-intercept form $y = mx + b$.
To do this, solve for y:

$$\begin{aligned} 3y &= -2x + 6 \\ y &= -\frac{2}{3}x + 2. \end{aligned}$$

It follows that the slope is $-\frac{2}{3}$ and y-intercept is $(0, 2)$. $\square$

The Point-Slope Form of the Equation of a Line

The Point-Slope Form of the Equation of a Line

The equation

$$y - y_0 = m(x - x_0)$$

is the equation of the line that passes through the point (x_0, y_0) and has slope equal to m.

An Application of the Point-Slope From

Example

Since the beginning of the year, the price of whole wheat bread at a local supermarket has been rising at a constant rate of 2 cents per month.
By June 1, the price has reached 1.46 € per loaf.
Express the price of the bread as a function of time and determine the price at the beginning of the year.

Solution...

- Denote by t the number of months since beginning of the year and by p the price of the bread (in cents).
- Since p changes at a constant rate with respect to t, the function relating p to t must be linear.

□

An Application of the Point-Slope From. (Continued)

. . . Solution. . .

- Since the price p increases by 2 each time t increases by 1, the slope of the line must be 2.
- The fact that the price was 146 cents (1.46 €) on June 1, implies that the line passes through the point $(5, 146)$.
- Use the point-slope formula

$$p - p_0 = m(t - t_0)$$

with $m = 2$, $t_0 = 5$, $p_0 = 146$, to get

$$\begin{aligned} p - 146 &= 2(t - 5) \\ p &= 2t + 136. \end{aligned}$$

□

An Application of the Point-Slope From. (Continued)

. . . Solution.

- The price at the beginning of the year is obtained for $t = 0$:

$$2 \cdot 0 + 136 = 136$$

  cents; i.e., 1.36 €.

- The corresponding line is shown in the following figure.

$\square$

An Application of the Point-Slope From. (Continued)

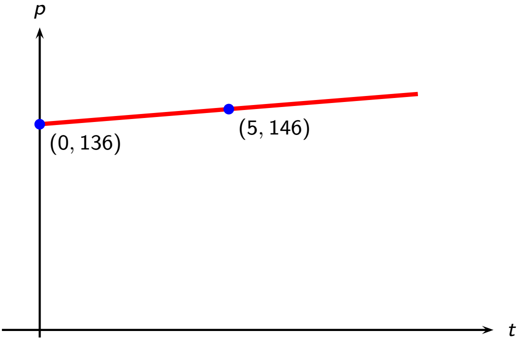

Figure: The rising price of bread: $p = 2t + 136$.

For Further Reading

- http://fberisha.netfirms.com
- Homework: Exercises from teaching materials
- L. D. Hofmann, G. L. Bradley, *Calculus – for business, economics and life sciences,* pp. 29–46
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks,* pp. 122–133

Summary

- Linear functions: constant rate of change
- Slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope-intercept formula: $y = mx + b$
- Point-slope formula: $y - y_0 = m(x - x_0)$