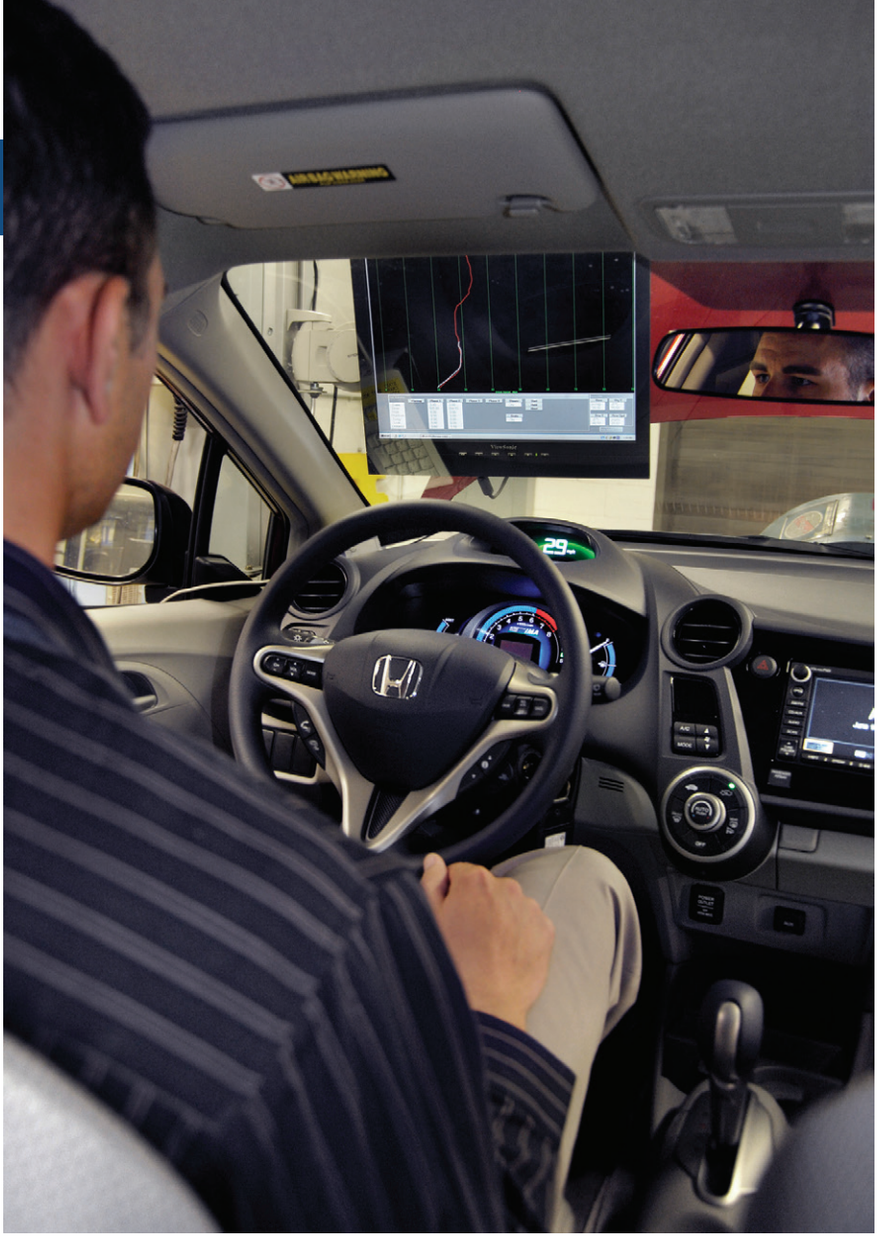
CHAPTER 8

Confidence Intervals



Learning Objectives

After mastering the material in this chapter, you will be able to:

- **LO8-1** Calculate and interpret a *z*-based confidence interval for a population mean when σ is known.
- **LO8-2** Describe the properties of the *t* distribution and use a *t* table.
- **LO8-3** Calculate and interpret a *t*-based confidence interval for a population mean when σ is unknown.
- **LO8-4** Determine the appropriate sample size when estimating a population mean.
- **LO8-5** Calculate and interpret a large sample confidence interval for a population proportion.
- **LO8-6** Determine the appropriate sample size when estimating a population proportion.
- **LO8-7** Find and interpret confidence intervals for parameters of finite populations (Optional).

Chapter Outline

- **8.1** *z*-Based Confidence Intervals for a Population Mean: σ Known
- **8.2** *t*-Based Confidence Intervals for a Population Mean: σ Unknown
- **8.3** Sample Size Determination
- **8.4** Confidence Intervals for a Population Proportion
- **8.5** Confidence Intervals for Parameters of Finite Populations (Optional)

We have seen that the sample mean is the point estimate of the population mean and the sample proportion is the point estimate of the population proportion. In general, although a point estimate is a reasonable one-number estimate of a population parameter (mean, proportion, or the like), the point estimate will not—unless we are extremely lucky—equal the true value of the population parameter.

In this chapter we study how to use a **confidence interval** to estimate a population parameter. A confidence interval for a population parameter is an interval, or range of numbers, constructed around the point estimate so that we are very sure, or confident, that the true value of the population parameter is inside the interval.

By computing such an interval, we estimate—with confidence—the possible values that a population parameter might equal. This, in turn, can help us to assess—with confidence—whether a particular business improvement has been made or is needed.

In order to illustrate confidence intervals, we revisit several cases introduced in earlier chapters and also introduce some new cases. For example:

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In the **Car Mileage Case,** we use a confidence interval to provide strong evidence that the mean EPA combined city and highway mileage for the automaker's new midsize model meets the tax credit standard of 31 mpg.

In the **e-billing Case,** we use a confidence interval to more completely assess the reduction in mean payment time that was achieved by the new billing system.

In the **Cheese Spread Case,** we use a confidence interval to provide strong evidence that fewer than 10 percent of all current purchasers will stop buying the cheese spread if the new spout is used, and, therefore, that it is reasonable to use the new spout.

8.1 *z*-Based Confidence Intervals for a Population Mean: σ Known

LO8-1 Calculate and interpret a *z*-based confidence interval for a population mean when σ is known.

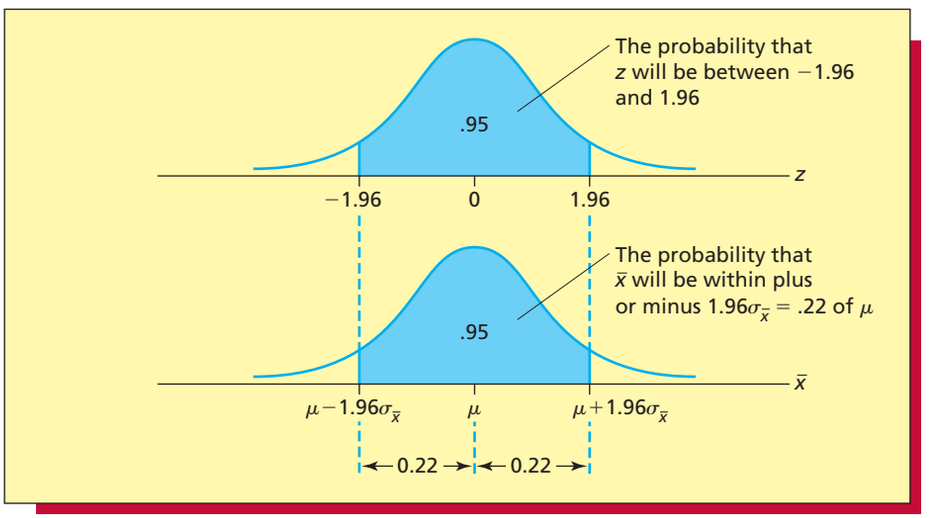
An introduction to confidence intervals for a population mean

In the *car mileage case*, we have seen that an automaker has introduced a new midsize model and wishes to estimate the mean EPA combined city and highway mileage, μ, that would be obtained by all cars of this type. In order to estimate μ, the automaker has conducted EPA mileage tests on a random sample of 50 of its new midsize cars and has obtained the sample of mileages in Table 1.6 (page 11). The mean of this sample of mileages, which is $\bar{x} = 31.56$ mpg, is the point estimate of μ. However, a sample mean will not—unless we are extremely lucky—equal the true value of a population mean. Therefore, the sample mean of 31.56 mpg does not, by itself, provide us with any confidence about the true value of the population mean μ. One way to estimate μ with confidence is to calculate a *confidence interval* for this mean.

A **confidence interval** for a population mean is an interval constructed around the sample mean so that we are reasonably sure, or confident, that this interval contains the population mean. Any confidence interval for a population mean is based on what is called a **confidence level.** This confidence level is a percentage (for example, 95 percent or 99 percent) that expresses how confident we are that the confidence interval contains the population mean. In order to explain the exact meaning of a confidence level, we will begin in the car mileage case by finding and interpreting a confidence interval for a population mean that is based on the most commonly used confidence level—the 95 percent level. Then we will generalize our discussion and show how to find and interpret a confidence interval that is based on any confidence level.

Before the automaker selected the sample of $n = 50$ new midsize cars and tested them as prescribed by the EPA, there were many samples of 50 cars and corresponding mileages that the automaker might have obtained. Because different samples generally have different sample means, we consider the probability distribution of the population of all possible sample means that would be obtained from all possible samples of $n = 50$ car mileages. In Chapter 7 we have seen that such a probability distribution is called the sampling distribution of the sample mean, and we have studied various properties of sampling distributions. Several of these properties tell us that, if the population of all individual midsize car mileages is normally distributed with mean μ and standard deviation σ,

FIGURE 8.1 The Sampling Distribution of the Sample Mean



then for any sample size n the sampling distribution of the sample mean is a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Moreover, because the sample size n is 50, and because we can assume that the true value of the population standard deviation σ is 0.8 mpg (as discussed on page 275 of Chapter 7), it follows that

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.8}{\sqrt{50}} = .113$$

This allows us to reason as follows:

1 In Chapter 6 (page 236) we have seen that the area under the standard normal curve between -1.96 and 1.96 is .95 (see Figure 8.1). This .95 area says that 95 percent of the values in any normally distributed population are within plus or minus 1.96 standard deviations of the mean of the population. Because the population of all possible sample means is normally distributed with mean μ and standard deviation $\sigma_{\bar{x}} = .113$, it follows that 95 percent of all possible sample means are within plus or minus $1.96\sigma_{\bar{x}} = 1.96(.113) = .22$ of the population mean μ. That is, **considered before we select the sample,** the probability is .95 that the sample mean $\bar{x}$ will be within plus or minus $1.96\sigma_{\bar{x}} = .22$ of the population mean μ (again, see Figure 8.1).

2 Saying

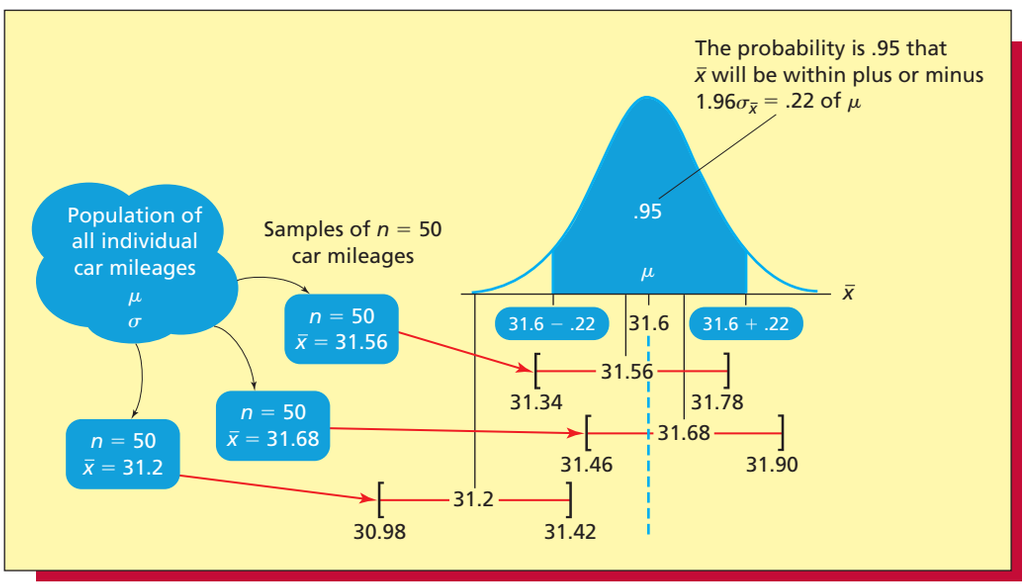
$$\bar{x} \text{ will be within plus or minus .22 of } \mu$$

is the same as saying

$$\bar{x} \text{ will be such that the interval } [\bar{x} \pm .22] \text{ contains } \mu$$

To understand this, consider Figure 8.2. This figure illustrates three possible samples of 50 mileages and the means of these samples. Also, this figure assumes that (unknown to any human being) the true value of the population mean μ is 31.6. Then, as illustrated in Figure 8.2, because the sample mean $\bar{x} = 31.56$ is within .22 of $\mu = 31.6$, the interval $[31.56 \pm .22] = [31.34, 31.78]$ contains μ. Similarly, because the sample mean $\bar{x} = 31.68$ is within .22 of $\mu = 31.6$, the interval $[31.68 \pm .22] = [31.46, 31.90]$ contains μ. However, because the sample mean $\bar{x} = 31.2$ is not within .22 of $\mu = 31.6$, the interval $[31.2 \pm .22] = [30.98, 31.42]$ does not contain μ.

FIGURE 8.2 Three 95 Percent Confidence Intervals for μ



3 In statement 1 we showed that the probability is .95 that the sample mean $\bar{x}$ will be within plus or minus $1.96\sigma_{\bar{x}} = .22$ of the population mean μ. In statement 2 we showed that $\bar{x}$ being within plus or minus .22 of μ is the same as the interval $[\bar{x} \pm .22]$ containing μ. Combining these results, we see that the probability is .95 that the sample mean $\bar{x}$ will be such that the interval

$$[\bar{x} \pm 1.96\sigma_{\bar{x}}] = [\bar{x} \pm .22]$$

contains the population mean μ.

A 95 percent confidence interval for μ Statement 3 says that, **before we randomly select the sample,** there is a .95 probability that we will obtain an interval $[\bar{x} \pm .22]$ that contains the population mean μ. In other words, 95 percent of all intervals that we might obtain contain μ, and 5 percent of these intervals do not contain μ. For this reason, we call the interval $[\bar{x} \pm .22]$ a **95 percent confidence interval for μ.** To better understand this interval, we must realize that, **when we actually select the sample,** we will observe one particular sample from the extremely large number of possible samples. Therefore, we will obtain one particular confidence interval from the extremely large number of possible confidence intervals. For example, recall that when the automaker randomly selected the sample of $n = 50$ cars and tested them as prescribed by the EPA, the automaker obtained the sample of 50 mileages given in Table 1.6. The mean of this sample is $\bar{x} = 31.56$ mpg, and a histogram constructed using this sample (see Figure 2.9 on page 46) indicates that the population of all individual car mileages is normally distributed. It follows that a 95 percent confidence interval for the population mean mileage μ of the new midsize model is

$$[\bar{x} \pm .22] = [31.56 \pm .22]$$
$$= [31.34, 31.78]$$

Because we do not know the true value of μ, we do not know for sure whether this interval contains μ. However, we are 95 percent confident that this interval contains μ. That is, we are 95 percent confident that μ is between 31.34 mpg and 31.78 mpg. What we mean by "95 percent confident" is that we hope that the confidence interval [31.34, 31.78] is one of the 95 percent of all confidence intervals that contain μ and not one of the 5 percent of all confidence intervals that do not contain μ. Here, we say that 95 percent is the **confidence level** associated with the confidence interval.

A practical application To see a practical application of the automaker's confidence interval, recall that the federal government will give a tax credit to any automaker selling a midsize model equipped with an automatic transmission that has an EPA combined city and highway mileage estimate of at least 31 mpg. Furthermore, to ensure that it does not overestimate a car model's mileage, the EPA will obtain the model's mileage estimate by rounding down—to the nearest mile per gallon—the lower limit of a 95 percent confidence interval for the model's mean mileage μ. That is, the model's mileage estimate is an estimate of the smallest that μ might reasonably be. When we round down the lower limit of the automaker's 95 percent confidence interval for μ, [31.34, 31.78], we find that the new midsize model's mileage estimate is 31 mpg. Therefore, the automaker will receive the tax credit.[1]

A general confidence interval procedure We will next present a general procedure for finding a confidence interval for a population mean μ. To do this, we assume that the sampled population is normally distributed, or the sample size n is large. Under these conditions, the sampling distribution of the sample mean $\bar{x}$ is exactly (or approximately, by the Central Limit Theorem) a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. In the previous subsection, we *started* with the normal points -1.96 and 1.96. Then we showed that, because the area under the standard normal curve between -1.96 and 1.96 is .95, the probability is .95 that the confidence interval $[\bar{x} \pm 1.96\sigma_{\bar{x}}]$ will contain the population mean. Usually, we do not start with two normal points, but rather we start by choosing the probability (for example, .95 or .99) that the confidence interval will contain the population mean. This probability is called the **confidence coefficient.** Next, we find the normal points that have a symmetrical area between them under the standard normal curve that is equal to the confidence coefficient. Then, using $\bar{x}$, $\sigma_{\bar{x}}$, and the normal points, we find the confidence interval that is based on the confidence coefficient. To illustrate this, we will start with a confidence coefficient of .95 and use the following three-step procedure to find the appropriate normal points and the corresponding 95 percent confidence interval for the population mean:

Step 1: As illustrated in Figure 8.3, place a symmetrical area of .95 under the standard normal curve and find the area in the normal curve tails beyond the .95 area. Because the entire area under the standard normal curve is 1, the area in both normal curve tails is $1-.95 = .05$, and the area in each tail is .025.

Step 2: Find the normal point $z_{.025}$ that gives a right-hand tail area under the standard normal curve equal to .025, and find the normal point $-z_{.025}$ that gives a left-hand tail area under the curve equal to .025. As shown in Figure 8.3, the area under the standard normal curve between $-z_{.025}$ and $z_{.025}$ is .95, and the area under this curve to the left of $z_{.025}$ is .975. Looking up a cumulative area of .975 in Table A.3 (see page 790) or in Table 8.1 (which shows a portion of Table A.3), we find that $z_{.025} = 1.96$.

Step 3: Form the following 95 percent confidence interval for the population mean:

$$[\bar{x} \pm z_{.025}\sigma_{\bar{x}}] = \left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$$

If all possible samples were used to calculate this interval, then 95 percent of the resulting intervals would contain the population mean. For example, recall in the car mileage case that $n = 50$, $\bar{x} = 31.56$, and $\sigma = .8$. Therefore, we can directly calculate the previously obtained 95 percent confidence interval for the midsize model's mean mileage μ as follows:

$$\begin{aligned}\left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right] &= \left[31.56 \pm 1.96\frac{.8}{\sqrt{50}}\right] \\ &= [31.56 \pm .22] \\ &= [31.34, 31.78]\end{aligned}$$

[1] This example is based on the authors' conversations with the EPA. However, there are approaches for showing that μ is at least 31 mpg that differ from the approach that uses the confidence interval [31.34, 31.78]. Some of these other approaches are discussed in Chapter 9, but the EPA tells the authors that it would use the confidence interval approach described here.

FIGURE 8.3 The Point $z_{.025}$

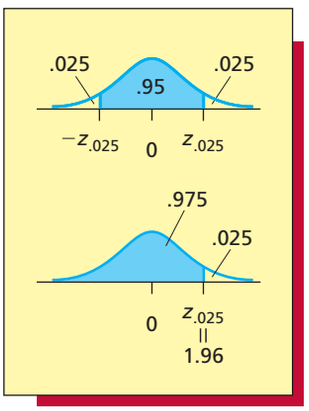


FIGURE 8.4 The Point $z_{.005}$

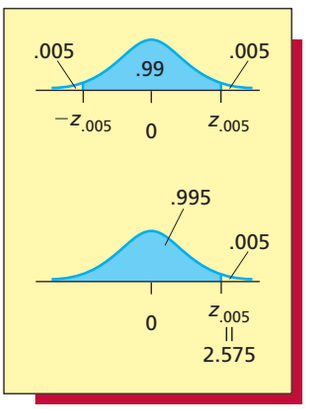


TABLE 8.1 Cumulative Areas under the Standard Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

We next start with a confidence coefficient of .99 and find the corresponding 99 percent confidence interval for the population mean:

Step 1: As illustrated in Figure 8.4, place a symmetrical area of .99 under the standard normal curve, and find the area in the normal curve tails beyond the .99 area. Because the entire area under the standard normal curve is 1, the area in both normal curve tails is $1 - .99 = .01$, and the area in each tail is .005.

Step 2: Find the normal point $z_{.005}$ that gives a right-hand tail area under the standard normal curve equal to .005, and find the normal point $-z_{.005}$ that gives a left-hand tail area under the curve equal to .005. As shown in Figure 8.4, the area under the standard normal curve between $-z_{.005}$ and $z_{.005}$ is .99, and the area under this curve to the left of $z_{.005}$ is .995. Looking up a cumulative area of .995 in Table A.3 (see page 790) or in Table 8.1, we find that $z_{.005} = 2.575$.

Step 3: Form the following 99 percent confidence interval for the population mean.

$$[\bar{x} \pm z_{.005}\sigma_{\bar{x}}] = \left[\bar{x} \pm 2.575\frac{\sigma}{\sqrt{n}}\right]$$

If all possible samples were used to calculate this interval, then 99 percent of the resulting intervals would contain the population mean. For example, in the car mileage case, a 99 percent confidence interval for the midsize model's mean mileage μ is:

$$\left[\bar{x} \pm 2.575\frac{\sigma}{\sqrt{n}}\right] = \left[31.56 \pm 2.575\frac{.8}{\sqrt{50}}\right]$$
$$= [31.56 \pm .29]$$
$$= [31.27, 31.85]$$

FIGURE 8.5 The Point $z_{\alpha/2}$

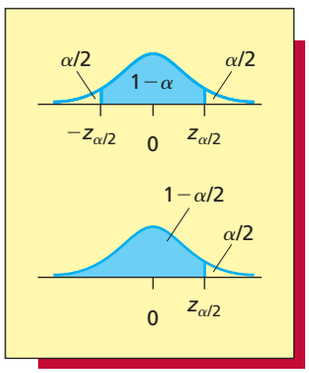


TABLE 8.2 The Normal Point $z_{\alpha/2}$ for Various Levels of Confidence

100(1 − α)%	1 − α	α	α/2	1 − α/2	$z_{\alpha/2}$
90%	.90	.10	.05	.95	$z_{.05} = 1.645$
95%	.95	.05	.025	.975	$z_{.025} = 1.96$
98%	.98	.02	.01	.99	$z_{.01} = 2.33$
99%	.99	.01	.005	.995	$z_{.005} = 2.575$

To compare the 95 percent confidence interval $[\bar{x} \pm 1.96(\sigma/\sqrt{n})]$ with the 99 percent confidence interval $[\bar{x} \pm 2.575(\sigma/\sqrt{n})]$, note that each of these confidence intervals can be expressed in the form [$\bar{x}$ ± margin of error]. Here, for a given level of confidence, the **margin of error** expresses the furthest that the sample mean $\bar{x}$ might be from the population mean μ. Moreover, the margin of error $2.575(\sigma/\sqrt{n})$ used to compute the 99 percent interval is larger than the margin of error $1.96(\sigma/\sqrt{n})$ used to compute the 95 percent interval. Therefore, the 99 percent interval is the longer of these intervals. **In general, increasing the confidence level (1) has the advantage of making us more confident that μ is contained in the confidence interval, but (2) has the disadvantage of increasing the margin of error and thus providing a less precise estimate of the true value of μ.** Frequently, 95 percent confidence intervals are used to make conclusions. If conclusions based on stronger evidence are desired, 99 percent intervals are sometimes used. In the car mileage case, the fairly large sample size of $n = 50$ produces a 99 percent margin of error, $2.575(.8/\sqrt{50}) = .29$, that is not much larger than the 95 percent margin of error, $1.96(.8/\sqrt{50}) = .22$. Therefore, the 99 percent confidence interval for μ, $[31.56 \pm .29] = [31.27, 31.85]$, is not much longer than the 95 percent confidence interval for μ, $[31.56 \pm .22] = [31.34, 31.78]$.

In general, we let α denote the probability that a confidence interval for a population mean will *not* contain the population mean. This implies that $1 - \alpha$ is the probability that the confidence interval will contain the population mean. In order to find a confidence interval for a population mean that is based on a confidence coefficient of $1 - \alpha$ (that is, a **100(1 − α) percent confidence interval** for the population mean), we do the following:

Step 1: As illustrated in Figure 8.5, place a symmetrical area of $1 - \alpha$ under the standard normal curve, and find the area in the normal curve tails beyond the $1 - \alpha$ area. Because the entire area under the standard normal curve is 1, the combined areas in the normal curve tails are α, and the area in each tail is $\alpha/2$.

Step 2: Find the normal point $z_{\alpha/2}$ that gives a right-hand tail area under the standard normal curve equal to $\alpha/2$, and find the normal point $-z_{\alpha/2}$ that gives a left-hand tail area under this curve equal to $\alpha/2$. As shown in Figure 8.5, the area under the standard normal curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is $(1-\alpha)$, and the area under this curve to the left of $z_{\alpha/2}$ is $1-\alpha/2$. **This implies that we can find $z_{\alpha/2}$ by looking up a cumulative area of $1-\alpha/2$ in Table A.3** (page 790).

Step 3: Form the following $100(1 - \alpha)$ percent confidence interval for the population mean.

$$[\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}}] = \left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

If all possible samples were used to calculate this interval, then $100(1 - \alpha)$ percent of the resulting intervals would contain the population mean. Moreover, we call **100(1 − α) percent** the **confidence level** associated with the confidence interval.

The general formula that we just obtained for a $100(1 - \alpha)$ percent confidence interval for a population mean implies that we now have a formal way to find the normal point corresponding to a particular level of confidence. Specifically, we can set $100(1 - \alpha)$ percent equal to the particular level of confidence, solve for α, and use a cumulative normal table to find the normal point $z_{\alpha/2}$ corresponding to the cumulative area $1 - \alpha/2$. For example, suppose that we wish to find a 99 percent confidence interval for the population mean. Then, because $100(1 - \alpha)$ percent equals 99 percent, it follows that $1-\alpha = .99$, $\alpha = .01$, $\alpha/2 = .005$, and $1 - \alpha/2 = .995$. Looking up .995 in a cumulative normal table, we find that $z_{\alpha/2} = z_{.005} = 2.575$. This normal point is the same normal point that we previously found using the three-step procedure and the normal curves illustrated in Figure 8.4. Table 8.2 summarizes finding the values of $z_{\alpha/2}$ for different values of the confidence level $100(1 - \alpha)$ percent.

The following box summarizes the formula used in calculating a $100(1 - \alpha)$ percent confidence interval for a population mean μ. This interval is based on the normal distribution and assumes that the true value of σ is known. If (as is usually the case) σ is not known, we can use a confidence interval for μ discussed in the next section.

A Confidence Interval for a Population Mean μ: σ Known

Suppose that the sampled population is normally distributed with mean μ and standard deviation σ. Then a $100(1 - \alpha)$ percent confidence interval for μ is

$$\left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = \left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

Here, $z_{\alpha/2}$ is the normal point that gives a right-hand tail area under the standard normal curve of $\alpha/2$. The normal point $z_{\alpha/2}$ can be found by looking up a cumulative area of $1 - \alpha/2$ in Table A.3 (page 790). This confidence interval is also approximately valid for non-normal populations if the sample size is large (at least 30).

EXAMPLE 8.1 The e-billing Case: Reducing Mean Bill Payment Time

Recall that a management consulting firm has installed a new computer-based electronic billing system in a Hamilton, Ohio, trucking company. The population mean payment time using the trucking company's old billing system was approximately equal to, but no less than, 39 days. In order to assess whether the population mean payment time, μ, using the new billing system is substantially less than 39 days, the consulting firm will use the sample of $n = 65$ payment times in Table 2.4 to find a 99 percent confidence interval for μ. The mean of the 65 payment times is $\bar{x} = 18.1077$ days, and we will assume that the true value of the population standard deviation σ for the new billing system is 4.2 days (as discussed on page 280 of Chapter 7). Then, because we previously showed that the normal point corresponding to 99 percent confidence is $z_{\alpha/2} = z_{.005} = 2.575$, a 99 percent confidence interval for μ is

$$\begin{aligned}\left[\bar{x} \pm z_{.005}\frac{\sigma}{\sqrt{n}}\right] &= \left[18.1077 \pm 2.575\frac{4.2}{\sqrt{65}}\right] \\ &= [18.1077 \pm 1.3414] \\ &= [16.8,\ 19.4]\end{aligned}$$

Recalling that the mean payment time using the old billing system is 39 days, this interval says that we are 99 percent confident that the population mean payment time using the new billing system is between 16.8 days and 19.4 days. Therefore, we are 99 percent confident that the new billing system reduces the mean payment time by at most $39 - 16.8 = 22.2$ days and by at least $39 - 19.4 = 19.6$ days.

Exercises for Section 8.1

CONCEPTS

connect

8.1 Explain why it is important to calculate a confidence interval.

8.2 Explain the meaning of the term "95 percent confidence."

8.3 Under what conditions is the confidence interval $[\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})]$ for μ valid?

8.4 For a fixed sample size, what happens to a confidence interval for μ when we increase the level of confidence?

8.5 For a fixed level of confidence, what happens to a confidence interval for μ when we increase the sample size?

METHODS AND APPLICATIONS

8.6 Suppose that, for a sample of size $n = 100$ measurements, we find that $\bar{x} = 50$. Assuming that σ equals 2, calculate confidence intervals for the population mean μ with the following confidence levels:

a 95% **b** 99% **c** 97% **d** 80% **e** 99.73% **f** 92%

8.7 THE TRASH BAG CASE DS TrashBag

Consider the trash bag problem. Suppose that an independent laboratory has tested trash bags and has found that no 30-gallon bags that are currently on the market have a mean breaking strength of 50 pounds or more. On the basis of these results, the producer of the new, improved trash bag feels sure that its 30-gallon bag will be the strongest such bag on the market if the new trash bag's mean breaking strength can be shown to be at least 50 pounds. The mean of the sample of 40 trash bag breaking strengths in Table 1.9 is $\bar{x} = 50.575$. If we let μ denote the mean of the breaking strengths of all possible trash bags of the new type and assume that σ equals 1.65:

a Calculate 95 percent and 99 percent confidence intervals for μ.

b Using the 95 percent confidence interval, can we be 95 percent confident that μ is at least 50 pounds? Explain.

c Using the 99 percent confidence interval, can we be 99 percent confident that μ is at least 50 pounds? Explain.

d Based on your answers to parts *b* and *c*, how convinced are you that the new 30-gallon trash bag is the strongest such bag on the market?

8.8 THE BANK CUSTOMER WAITING TIME CASE DS WaitTime

Recall that a bank manager has developed a new system to reduce the time customers spend waiting to be served by tellers during peak business hours. The mean waiting time during peak business hours under the current system is roughly 9 to 10 minutes. The bank manager hopes that the new system will have a mean waiting time that is less than six minutes. The mean of the sample of 100 bank customer waiting times in Table 1.8 is $\bar{x} = 5.46$. If we let μ denote the mean of all possible bank customer waiting times using the new system and assume that σ equals 2.47:

a Calculate 95 percent and 99 percent confidence intervals for μ.

b Using the 95 percent confidence interval, can the bank manager be 95 percent confident that μ is less than six minutes? Explain.

c Using the 99 percent confidence interval, can the bank manager be 99 percent confident that μ is less than six minutes? Explain.

d Based on your answers to parts *b* and *c*, how convinced are you that the new mean waiting time is less than six minutes?

8.9 THE VIDEO GAME SATISFACTION RATING CASE DS VideoGame

The mean of the sample of 65 customer satisfaction ratings in Table 1.7 is $\bar{x} = 42.95$. If we let μ denote the mean of all possible customer satisfaction ratings for the XYZ Box video game system, and assume that σ equals 2.64:

a Calculate 95 percent and 99 percent confidence intervals for μ.

b Using the 95 percent confidence interval, can we be 95 percent confident that μ is at least 42 (recall that a very satisfied customer gives a rating of at least 42)? Explain.

c Using the 99 percent confidence interval, can we be 99 percent confident that μ is at least 42? Explain.

d Based on your answers to parts *b* and *c*, how convinced are you that the mean satisfaction rating is at least 42?

8.10 In an article in *Marketing Science,* Silk and Berndt investigate the output of advertising agencies. They describe ad agency output by finding the shares of dollar billing volume coming from various media categories such as network television, spot television, newspapers, radio, and so forth.

a Suppose that a random sample of 400 U.S. advertising agencies gives an average percentage share of billing volume from network television equal to 7.46 percent, and assume that σ equals 1.42 percent. Calculate a 95 percent confidence interval for the mean percentage share of billing volume from network television for the population of all U.S. advertising agencies.

b Suppose that a random sample of 400 U.S. advertising agencies gives an average percentage share of billing volume from spot television commercials equal to 12.44 percent, and assume that σ equals 1.55 percent. Calculate a 95 percent confidence interval for the mean percentage share of billing volume from spot television commercials for the population of all U.S. advertising agencies.

c Compare the confidence intervals in parts *a* and *b*. Does it appear that the mean percentage share of billing volume from spot television commercials for U.S. advertising agencies is greater than the mean percentage share of billing volume from network television? Explain.

8.11 In an article in *Accounting and Business Research,* Carslaw and Kaplan investigate factors that influence "audit delay" for firms in New Zealand. Audit delay, which is defined to be the length of time (in days) from a company's financial year-end to the date of the auditor's report, has been found to affect the market reaction to the report. This is because late reports often seem to be associated with lower returns and early reports often seem to be associated with higher returns.

Carslaw and Kaplan investigated audit delay for two kinds of public companies—owner-controlled and manager-controlled companies. Here a company is considered to be owner controlled if 30 percent or more of the common stock is controlled by a single outside investor (an investor not part of the management group or board of directors). Otherwise, a company is considered manager controlled. It was felt that the type of control influences audit delay. To quote Carslaw and Kaplan:

> Large external investors, having an acute need for timely information, may be expected to pressure the company and auditor to start and to complete the audit as rapidly as practicable.

a Suppose that a random sample of 100 public owner-controlled companies in New Zealand is found to give a mean audit delay of $\bar{x} = 82.6$ days, and assume that σ equals 33 days. Calculate a 95 percent confidence interval for the population mean audit delay for all public owner-controlled companies in New Zealand.

b Suppose that a random sample of 100 public manager-controlled companies in New Zealand is found to give a mean audit delay of $\bar{x} = 93$ days, and assume that σ equals 37 days. Calculate a 95 percent confidence interval for the population mean audit delay for all public manager-controlled companies in New Zealand.

c Use the confidence intervals you computed in parts *a* and *b* to compare the mean audit delay for all public owner-controlled companies versus that of all public manager-controlled companies. How do the means compare? Explain.

8.12 In an article in the *Journal of Marketing,* Bayus studied the differences between "early replacement buyers" and "late replacement buyers" in making consumer durable good replacement purchases. Early replacement buyers are consumers who replace a product during the early part of its lifetime, while late replacement buyers make replacement purchases late in the product's lifetime. In particular, Bayus studied automobile replacement purchases. Consumers who traded in cars with ages of zero to three years and mileages of no more than 35,000 miles were classified as early replacement buyers. Consumers who traded in cars with ages of seven or more years and mileages of more than 73,000 miles were classified as late replacement buyers. Bayus compared the two groups of buyers with respect to demographic variables such as income, education, age, and so forth. He also compared the two groups with respect to the amount of search activity in the replacement purchase process. Variables compared included the number of dealers visited, the time spent gathering information, and the time spent visiting dealers.

a Suppose that a random sample of 800 early replacement buyers yields a mean number of dealers visited of $\bar{x} = 3.3$, and assume that σ equals .71. Calculate a 99 percent confidence interval for the population mean number of dealers visited by early replacement buyers.

b Suppose that a random sample of 500 late replacement buyers yields a mean number of dealers visited of $\bar{x} = 4.3$, and assume that σ equals .66. Calculate a 99 percent confidence interval for the population mean number of dealers visited by late replacement buyers.

c Use the confidence intervals you computed in parts *a* and *b* to compare the mean number of dealers visited by early replacement buyers with the mean number of dealers visited by late replacement buyers. How do the means compare? Explain.

LO8-2 Describe the properties of the *t* distribution and use a *t* table.

8.2 *t*-Based Confidence Intervals for a Population Mean: σ Unknown

If we do not know σ (which is usually the case), we can use the sample standard deviation s to help construct a confidence interval for μ. The interval is based on the sampling distribution of

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

If the sampled population is normally distributed, then for any sample size n this sampling distribution is what is called a ***t* distribution.**

The curve of the t distribution has a shape similar to that of the standard normal curve. Two t curves and a standard normal curve are illustrated in Figure 8.6. A t curve is symmetrical about zero, which is the mean of any t distribution. However, the t distribution is more spread out, or variable, than the standard normal distribution. Because the t statistic above is a function of two random variables, $\bar{x}$ and s, it is logical that the sampling distribution of this statistic is more variable than the sampling distribution of the z statistic, which is a function of only one random variable, $\bar{x}$. The exact spread, or standard deviation, of the t distribution depends on a parameter that is called the **number of degrees of freedom (denoted *df*).** The number of degrees of freedom df varies depending on the problem. In the present situation the sampling distribution of t has a number of degrees of freedom that equals the sample size minus 1. We say that this sampling distribution is a ***t* distribution with $n - 1$ degrees of freedom.** As the sample size n (and thus the number of degrees of freedom) increases, the spread of the t distribution decreases (see Figure 8.6). Furthermore, as the number of degrees of freedom approaches infinity, the curve of the t distribution approaches (that is, becomes shaped more and more like) the curve of the standard normal distribution.

In order to use the t distribution, we employ a ***t* point that is denoted t_α.** As illustrated in Figure 8.7, **t_α is the point on the horizontal axis under the curve of the *t* distribution that gives a right-hand tail area equal to α.** The value of t_α in a particular situation depends upon the right-hand tail area α and the number of degrees of freedom of the t distribution. Values of t_α are tabulated in a ***t* table.** Such a table is given in Table A.4 of Appendix A (pages 792 and 793) and a portion of Table A.4 is reproduced in this chapter as Table 8.3. In this t table, the rows correspond to the different numbers of degrees of freedom (which are denoted as df). The values of df are listed down the left side of the table, while the columns designate the right-hand tail area α. For example, suppose we wish to find the t point that gives a right-hand tail area of .025 under a t curve having $df = 14$ degrees of freedom. To do this, we look in Table 8.3 at the row labeled 14 and the column labeled $t_{.025}$. We find that this $t_{.025}$ point is 2.145 (also see Figure 8.8). Similarly, when there are $df = 14$ degrees of freedom, we find that $t_{.005} = 2.977$ (see Table 8.3 and Figure 8.9).

FIGURE 8.6 As the Number of Degrees of Freedom Increases, the Spread of the *t* Distribution Decreases and the *t* Curve Approaches the Standard Normal Curve

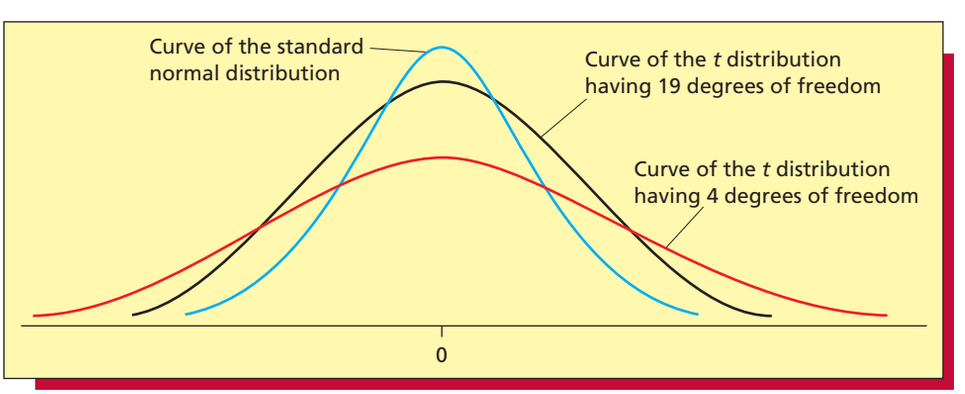


FIGURE 8.7 An Example of a t Point Giving a Specified Right-Hand Tail Area (This t Point Gives a Right-Hand Tail Area Equal to α).

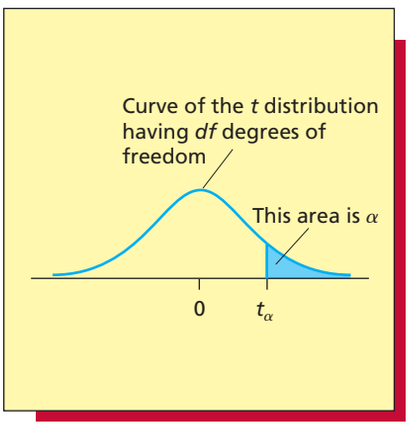


FIGURE 8.8 The t Point Giving a Right-Hand Tail Area of .025 under the t Curve Having 14 Degrees of Freedom: $t_{.025} = 2.145$

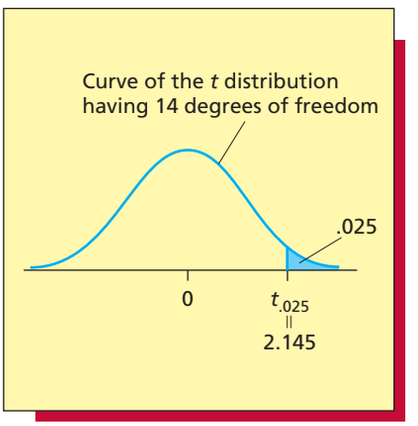
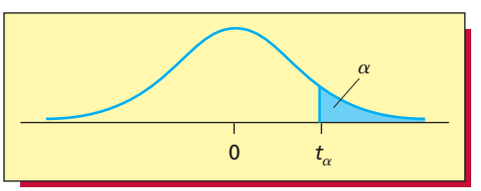


TABLE 8.3 A t Table



df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: E. S. Pearson and H. O. Hartley eds., *The Biometrika Tables for Statisticians* 1, 3d ed. (Biometrika, 1966). Reproduced by permission of Oxford University Press Biometrika Trustees.

FIGURE 8.9 **The t Point Giving a Right-Hand Tail Area of .005 Under the t Curve Having 14 Degrees of Freedom: $t_{.005} = 2.977$**

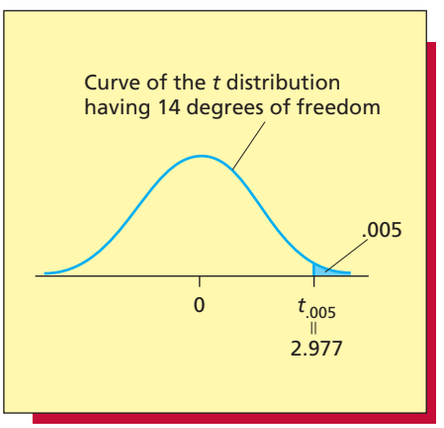
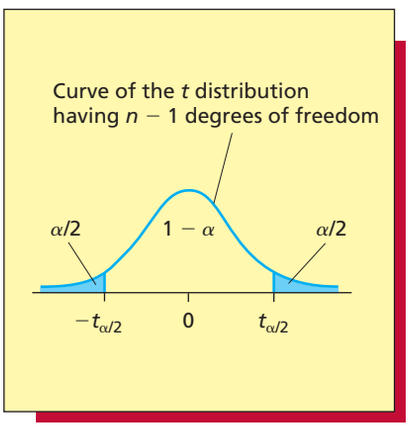


FIGURE 8.10 **The Point $t_{\alpha/2}$ with $n - 1$ Degrees of Freedom**



LO8-3 Calculate and interpret a t-based confidence interval for a population mean when σ is unknown.

Table 8.3 gives t points for degrees of freedom df from 1 to 30. The table also gives t points for 40, 60, 120, and an infinite number of degrees of freedom. Looking at this table, it is useful to realize that the normal points giving the various right-hand tail areas are listed in the row of the t table corresponding to an infinite (∞) number of degrees of freedom. Looking at the row corresponding to ∞, we see that, for example, $z_{.025} = 1.96$. Therefore, we can use this row in the t table as an alternative to using the normal table when we need to find normal points (such as $z_{\alpha/2}$ in Section 8.1).

Table A.4 of Appendix A (pages 792 and 793) gives t points for values of df from 1 to 100. We can use a computer to find t points based on values of df greater than 100. Alternatively, because a t curve based on more than 100 degrees of freedom is approximately the shape of the standard normal curve, t points based on values of df greater than 100 can be approximated by their corresponding z points. That is, when performing hand calculations, it is reasonable to approximate values of t_α by z_α when df is greater than 100.

We now present the formula for a $100(1 - \alpha)$ percent confidence interval for a population mean μ based on the t distribution.

A t-Based Confidence Interval for a Population Mean μ: σ Unknown

If the sampled population is normally distributed with mean μ, then a **$100(1 - \alpha)$ percent confidence interval for μ** is

$$\left[\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}\right]$$

Here s is the sample standard deviation, $t_{\alpha/2}$ is the t point giving a right-hand tail area of $\alpha/2$ under the t curve having $n - 1$ degrees of freedom, and n is the sample size. This confidence interval is also approximately valid for non-normal populations if the sample size is large (at least 30).

Before presenting an example, we need to make a few comments. First, it has been shown that, even if the sample size is not large, this confidence interval is approximately valid for many populations that are not exactly normally distributed. In particular, this interval is approximately valid for a mound-shaped, or single-peaked, population, even if the population is somewhat skewed to the right or left. Second, this interval employs the point $t_{\alpha/2}$, which as shown in Figure 8.10, gives a right-hand tail area equal to $\alpha/2$ under the t curve having $n - 1$ degrees of freedom. Here $\alpha/2$ is determined from the desired confidence level $100(1 - \alpha)$ percent.

EXAMPLE 8.2 The Commercial Loan Case: Mean Debt-to-Equity Ratio

One measure of a company's financial health is its *debt-to-equity ratio.* This quantity is defined to be the ratio of the company's corporate debt to the company's equity. If this ratio is too high, it is one indication of financial instability. For obvious reasons, banks often monitor the financial health of companies to which they have extended commercial loans. Suppose that, in order to reduce risk, a large bank has decided to initiate a policy limiting the mean debt-to-equity ratio for its portfolio of commercial loans to being less than 1.5. In order to estimate the mean debt-to-equity ratio of its (current) commercial loan portfolio, the bank randomly selects a sample of 15 of its commercial loan accounts. Audits of these companies result in the following debt-to-equity ratios:

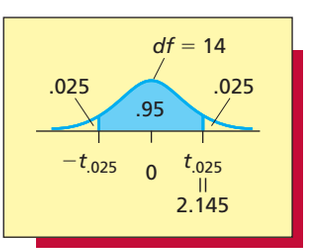
DebtEq

1.31	1.05	1.45	1.21	1.19
1.78	1.37	1.41	1.22	1.11
1.46	1.33	1.29	1.32	1.65

A stem-and-leaf display of these ratios is given on the page margin and looks reasonably bell-shaped and symmetrical. Furthermore, the sample mean and standard deviation of the ratios can be calculated to be $\bar{x} = 1.3433$ and $s = .1921$.

1.0	5
1.1	1 9
1.2	1 2 9
1.3	1 2 3 7
1.4	1 5 6
1.5	
1.6	5
1.7	8

Suppose the bank wishes to calculate a 95 percent confidence interval for the loan portfolio's mean debt-to-equity ratio, μ. The reasonably bell-shaped and symmetrical stem-and-leaf display in the page margin implies that the population of all debt-to-equity ratios is (approximately) normally distributed. Thus, we can base the confidence interval on the t-distribution. Because the bank has taken a sample of size $n = 15$, we have $n - 1 = 15 - 1 = 14$ degrees of freedom, and the level of confidence $100(1 - \alpha)\% = 95\%$ implies that $1 - \alpha = .95$ and $\alpha = .05$. Therefore, we use the t point $t_{\alpha/2} = t_{.05/2} = t_{.025}$, which is the t point giving a right-hand tail area of .025 under the t curve having 14 degrees of freedom. This t point is illustrated in the figure below:

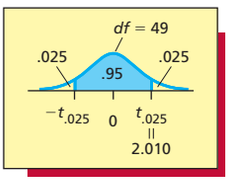


Using Table 8.3 (page 311), we find that $t_{.025}$ with 14 degrees of freedom is 2.145. It follows that the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[1.3433 \pm 2.145\frac{.1921}{\sqrt{15}}\right]$$
$$= [1.3433 \pm 0.1064]$$
$$= [1.2369, 1.4497]$$

This interval says the bank is 95 percent confident that the mean debt-to-equity ratio for its portfolio of commercial loan accounts is between 1.2369 and 1.4497. Based on this interval, the bank has strong evidence that the portfolio's mean ratio is less than 1.5 (or that the bank is in compliance with its new policy).

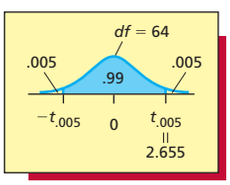
Recall that in the two cases discussed in Section 8.1 we calculated z-based confidence intervals for μ by assuming that the population standard deviation σ is known. If σ is actually not known (which would probably be true), we should compute t-based confidence intervals. Furthermore, recall that in each of these cases the sample size is large (at least 30). As stated in the summary box, **if the sample size is large, the t-based confidence interval for μ is approximately valid even if the sampled population is not normally distributed.** Therefore, consider the car mileage case and the sample of 50 mileages in Table 1.6, which has mean $\bar{x} = 31.56$ and



standard deviation $s = .7977$. The 95 percent t-based confidence interval for the population mean mileage μ of the new midsize model is

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[31.56 \pm 2.010\frac{.7977}{\sqrt{50}}\right] = [31.33, 31.79]$$

where $t_{.025} = 2.010$ is based on $n - 1 = 50 - 1 = 49$ degrees of freedom—see Table A.4 (page 792). This interval says we are 95 percent confident that the model's population mean mileage μ is between 31.33 mpg and 31.79 mpg. Based on this interval, the model's EPA mileage estimate is 31 mpg, and the automaker will receive the tax credit.

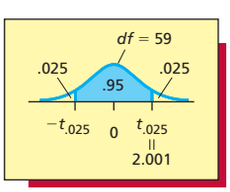


As another example, the sample of 65 payment times in Table 2.4 has mean $\bar{x} = 18.1077$ and standard deviation $s = 3.9612$. The 99 percent t-based confidence interval for the population mean payment time using the new electronic billing system is

$$\left[\bar{x} \pm t_{.005}\frac{s}{\sqrt{n}}\right] = \left[18.1077 \pm 2.655\frac{3.9612}{\sqrt{65}}\right] = [16.8, 19.4]$$

where $t_{.005} = 2.655$ is based on $n - 1 = 65 - 1 = 64$ degrees of freedom—see Table A.4 (page 792). Recalling that the mean payment time using the old billing system is 39 days, the interval says that we are 99 percent confident that the population mean payment time using the new billing system is between 16.8 days and 19.4 days. Therefore, we are 99 percent confident that the new billing system reduces the mean payment time by at most 22.2 days and by at least 19.6 days.

EXAMPLE 8.3 The Marketing Research Case: Rating A Bottle Design



Recall that a brand group is considering a new bottle design for a popular soft drink and that Table 1.5 (page 10) gives a random sample of $n = 60$ consumer ratings of this new bottle design. Let μ denote the mean rating of the new bottle design that would be given by all consumers. In order to assess whether μ exceeds the minimum standard composite score of 25 for a successful bottle design, the brand group will calculate a 95 percent confidence interval for μ. The mean and the standard deviation of the 60 bottle design ratings are $\bar{x} = 30.35$ and $s = 3.1073$. It follows that a 95 percent confidence interval for μ is

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[30.35 \pm 2.001\frac{3.1073}{\sqrt{60}}\right] = [29.5, 31.2]$$

where $t_{.025} = 2.001$ is based on $n - 1 = 60 - 1 = 59$ degrees of freedom—see Table A.4 (page 792). Because the interval says we are 95 percent confident that the population mean rating of the new bottle design is between 29.5 and 31.2, we are 95 percent confident that this mean rating exceeds the minimum standard of 25 by at least 4.5 points and by at most 6.2 points.

Confidence intervals for μ can be computed using Excel and MINITAB. For example, the MINITAB output in Figure 8.11 tells us that the t-based 95 percent confidence interval for the mean debt-to-equity ratio is [1.2370, 1.4497]. This result is, within rounding, the same interval calculated in Example 8.2. The MINITAB output also gives the sample mean $\bar{x} = 1.3433$, as well as the sample standard deviation $s = .1921$ and the quantity $s/\sqrt{n} = .0496$, which is called the **standard error of the estimate** $\bar{x}$ and denoted "SE Mean" on the MINITAB output. Finally, the MINITAB output gives a box plot of the sample of 15 debt-to-equity ratios and graphically illustrates under the box plot the 95 percent confidence interval for the mean debt-to-equity ratio. Figure 8.12 gives the Excel output of the information needed to calculate the t-based 95 percent confidence interval for the mean debt-to-equity ratio. If we consider the Excel output, we see that $\bar{x} = 1.3433$ (see "Mean"), $s = .1921$ (see "Standard Deviation"), $s/\sqrt{n} = .0496$ (see "Standard Error"), and $t_{.025}(s/\sqrt{n}) = .1064$ [see "Confidence Level (95.0%)"]. The interval, which must be hand calculated (or calculated by using Excel cell formulas), is $[1.3433 \pm .1064] = [1.2369, 1.4497]$.

FIGURE 8.11 MINITAB Output of a t-Based 95 Percent Confidence Interval for the Mean Debt-to-Equity Ratio

Variable	N	Mean	StDev	SE Mean	95% CI
Ratio	15	1.3433	0.1921	0.0496	(1.2370, 1.4497)

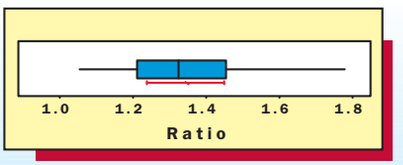
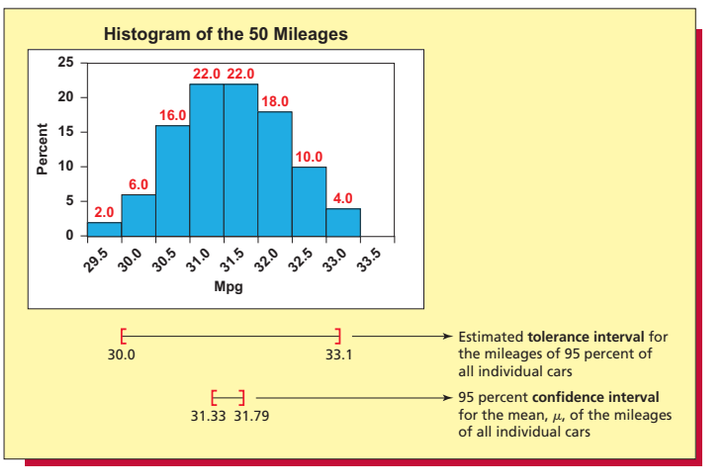


FIGURE 8.12 The Excel Output for the Debt-to-Equity Ratio Example

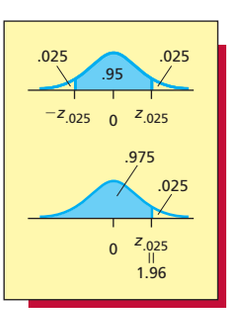
	A	B	C
1	*Descriptive Statistics*		
2			
3	Mean	1.3433	
4	Standard Error	0.0496	
5	Median	1.32	
6	Mode	#N/A	
7	Standard Deviation	0.1921	
8	Sample Variance	0.0369	
9	Kurtosis	0.8334	
10	Skewness	0.8050	
11	Range	0.73	
12	Minimum	1.05	
13	Maximum	1.78	
14	Sum	20.15	
15	Count	15	
16	Confidence Level(95.0%)	0.1064	

FIGURE 8.13 A Comparison of a Confidence Interval and a Tolerance Interval



Finally, if the sample size n is small and the sampled population is not mound-shaped or is highly skewed, then the t-based confidence interval for the population mean might not be valid. In this case we can use a **nonparametric method**—a method that makes no assumption about the shape of the sampled population and is valid for any sample size. In Chapter 18 we discuss nonparametric methods.

Making the correct interpretations: The difference between a confidence interval and a tolerance interval Recall in the car mileage case that the mean and the standard deviation of the sample of 50 mileages are $\bar{x} = 31.56$ and $s = .7977$. Also, we have seen on the previous page that a 95 percent t-based confidence interval for the mean, μ, of the mileages of all individual cars is $[\bar{x} \pm 2.010\,(s/\sqrt{50})] = [31.33, 31.79]$. A correct interpretation of this confidence interval says that we are 95 percent confident that the population mean mileage μ is between 31.33 mpg and 31.79 mpg. An incorrect interpretation—and one that is sometimes made by beginning statistics students—is that we estimate that 95 percent of all individual cars would (if tested) get between 31.33 mpg and 31.79 mpg. In general, an interval that contains a specified percentage of the individual measurements in a population is a *tolerance interval* (as previously discussed in Chapter 3). A tolerance interval is of the form $[\mu \pm z_{\alpha/2}\sigma]$ if the population of all individual measurements is normally distributed. For example, the histogram in Figure 8.13 suggests that the population of the mileages of all individual cars is normally distributed. Therefore, consider estimating a tolerance interval that contains the mileages of 95 percent of all individual cars. To do this, we first find the normal point $z_{\alpha/2}$ such that the area under the standard normal curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is .95. As shown in the page margin, the appropriate normal point $z_{\alpha/2}$ is $z_{.025} = 1.96$. Estimating μ and σ in the tolerance interval $[\mu \pm 1.96\sigma]$ by $\bar{x} = 31.56$ and $s = .7977$, we then obtain an estimated tolerance interval of $[31.56 \pm 1.96(.7977)] = [30.0, 33.1]$. This estimated tolerance interval implies that approximately 95 percent of all individual cars would (if tested) get between 30.0 mpg and 33.1 mpg. Furthermore, Figure 8.13 shows that the estimated tolerance interval, which is meant to contain the *many mileages* that would be obtained by 95 percent of all individual cars, is much longer than the 95 percent confidence interval, which is meant to contain the *single population mean* μ.



Exercises for Section 8.2

connect

CONCEPTS

8.13 Explain how each of the following changes as *the number of degrees of freedom* describing a t curve *increases:*

a The standard deviation of the t curve. **b** The points t_{α} and $t_{\alpha/2}$.

8.14 Discuss when it is appropriate to use the t-based confidence interval for μ.

METHODS AND APPLICATIONS

8.15 Using Table A.4 (page 792), find $t_{.100}$, $t_{.025}$, and $t_{.001}$ based on 11 degrees of freedom. Also, find these t points based on 6 degrees of freedom.

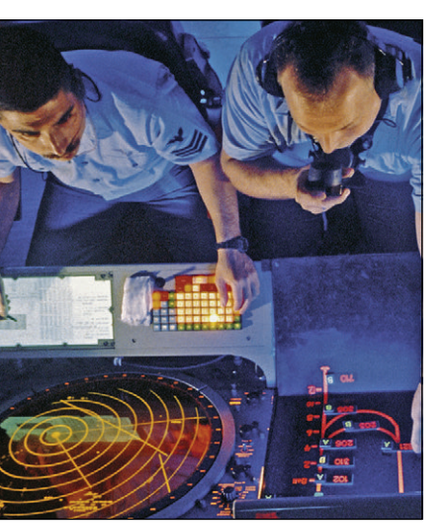
8.16 Suppose that for a sample of $n = 11$ measurements, we find that $\bar{x} = 72$ and $s = 5$. Assuming normality, compute confidence intervals for the population mean μ with the following levels of confidence:

a 95% **b** 99% **c** 80% **d** 90% **e** 98% **f** 99.8%

8.17 The *bad debt ratio* for a financial institution is defined to be the dollar value of loans defaulted divided by the total dollar value of all loans made. Suppose a random sample of seven Ohio banks is selected and that the bad debt ratios (written as percentages) for these banks are 7 percent, 4 percent, 6 percent, 7 percent, 5 percent, 4 percent, and 9 percent. Assuming the bad debt ratios are approximately normally distributed, the MINITAB output of a 95 percent confidence interval for the mean bad debt ratio of all Ohio banks is given below. Using the $\bar{x}$ and s on the MINITAB output, demonstrate the calculation of the 95 percent confidence interval, and calculate a 99 percent confidence interval for the mean debt-to-equity ratio. DS BadDebt

```
Variable   N      Mean     StDev   SE Mean          95% CI
D-Ratio    7   6.00000   1.82574   0.69007   (4.31147, 7.68853)
```

8.18 In Exercise 8.17, suppose bank officials claim that the mean bad debt ratio for all banks in the Midwest region is 3.5 percent and that the mean bad debt ratio for Ohio banks is higher. Using the 95 percent confidence interval (given by MINITAB), can we be 95 percent confident that this claim is true? Using the 99 percent confidence interval you calculated, can we be 99 percent confident that this claim is true?



8.19 **THE AIR SAFETY CASE**

Air traffic controllers have the crucial task of ensuring that aircraft don't collide. To do this, they must quickly discern when two planes are about to enter the same air space at the same time. They are aided by video display panels that track the aircraft in their sector and alert the controller when two flight paths are about to converge. The display panel currently in use has a mean "alert time" of 15 seconds. (The alert time is the time elapsing between the instant when two aircraft enter into a collision course and when a controller initiates a call to reroute the planes.) According to Ralph Rudd, a supervisor of air traffic controllers at the Greater Cincinnati International Airport, a new display panel has been developed that uses artificial intelligence to project a plane's current flight path into the future. This new panel provides air traffic controllers with an earlier warning that a collision is likely. It is hoped that the mean "alert time," μ, for the new panel is less than 8 seconds. In order to test the new panel, 15 randomly selected air traffic controllers are trained to use the panel and their alert times for a simulated collision course are recorded. The sample alert times (in seconds) are: 7.2, 7.5, 8.0, 6.8, 7.2, 8.4, 5.3, 7.3, 7.6, 7.1, 9.4, 6.4, 7.9, 6.2, 8.7. DS AlertTimes

a Using the fact that $\bar{x} = 7.4$ and $s = 1.026$, find a 95 percent confidence interval for the population mean alert time, μ, for the new panel.

b Can we be 95 percent confident that μ is less than 8 seconds?

8.20 Whole Foods is an all-natural grocery chain that has 50,000 square foot stores, up from the industry average of 34,000 square feet. Sales per square foot of supermarkets average just under $400 per square foot, as reported by *USA Today* in an article on "A whole new ballgame in grocery shopping." Suppose that sales per square foot in the most recent fiscal year are recorded for a random sample of 10 Whole Foods supermarkets. The data (sales dollars per square foot) are as follows: 854, 858, 801, 892, 849, 807, 894, 863, 829, 815. Using the fact that $\bar{x} = 846.2$ and $s = 32.866$, find a 95 percent confidence interval for the population mean sales dollars per square foot for all Whole Foods supermarkets during the most recent fiscal year. Are we 95 percent confident that this mean is greater than $800, the historical average for Whole Foods? DS WholeFoods

FIGURE 8.14 Excel Output for Exercise 8.21

	A	B	C
1	*Descriptive Statistics for Yield*		
2			
3	Mean	811	
4	Standard Error	8.7864	
5	Median	814	
6	Mode	#N/A	
7	Standard Deviation	19.6469	
8	Sample Variance	386	
9	Kurtosis	-0.1247	
10	Skewness	-0.2364	
11	Range	52	
12	Minimum	784	
13	Maximum	836	
14	Sum	4055	
15	Count	5	
16	Confidence Level(95.0%)	24.3948	

FIGURE 8.15 Excel Output for Exercise 8.24

	A	B	C
1	*Descriptive Statistics for Strength*		
2			
3	Mean	50.575	
4	Standard Error	0.2599	
5	Median	50.65	
6	Mode	50.9	
7	Standard Deviation	1.6438	
8	Sample Variance	2.7019	
9	Kurtosis	-0.2151	
10	Skewness	-0.0549	
11	Range	7.2	
12	Minimum	46.8	
13	Maximum	54	
14	Sum	2023	
15	Count	40	
16	Confidence Level(95.0%)	0.5257	

8.21 A production supervisor at a major chemical company wishes to determine whether a new catalyst, catalyst XA-100, increases the mean hourly yield of a chemical process beyond the current mean hourly yield, which is known to be roughly equal to, but no more than, 750 pounds per hour. To test the new catalyst, five trial runs using catalyst XA-100 are made. The resulting yields for the trial runs (in pounds per hour) are 801, 814, 784, 836, and 820. Assuming that all factors affecting yields of the process have been held as constant as possible during the test runs, it is reasonable to regard the five yields obtained using the new catalyst as a random sample from the population of all possible yields that would be obtained by using the new catalyst. Furthermore, we will assume that this population is approximately normally distributed. DS ChemYield

a Using the Excel output in Figure 8.14, find a 95 percent confidence interval for the mean of all possible yields obtained using catalyst XA-100.

b Based on the confidence interval, can we be 95 percent confident that the mean yield using catalyst XA-100 exceeds 750 pounds per hour? Explain.

8.22 **THE VIDEO GAME SATISFACTION RATING CASE** DS VideoGame

The mean and the standard deviation of the sample of $n = 65$ customer satisfaction ratings in Table 1.7 are $\bar{x} = 42.95$ and $s = 2.6424$. Calculate a t-based 95 percent confidence interval for μ, the mean of all possible customer satisfaction ratings for the XYZ-Box video game system. Are we 95 percent confident that μ is at least 42, the minimal rating given by a very satisfied customer?

8.23 **THE BANK CUSTOMER WAITING TIME CASE** DS WaitTime

The mean and the standard deviation of the sample of 100 bank customer waiting times in Table 1.8 are $\bar{x} = 5.46$ and $s = 2.475$. Calculate a t-based 95 percent confidence interval for μ, the mean of all possible bank customer waiting times using the new system. Are we 95 percent confident that μ is less than six minutes?

8.24 **THE TRASH BAG CASE** DS TrashBag

The mean and the standard deviation of the sample of 40 trash bag breaking strengths in Table 1.9 are $\bar{x} = 50.575$ and $s = 1.6438$.

a Use the Excel output in Figure 8.15 to calculate a t-based 95 percent confidence interval for μ, the mean of the breaking strengths of all possible new trash bags. Are we 95 percent confident that μ is at least 50 pounds?

b Assuming that the population of all individual trash bag breaking strengths is normally distributed, estimate tolerance intervals of the form $[\mu \pm z_{\alpha/2}\sigma]$ that contain (1) 95 percent of all individual trash bag breaking strengths and (2) 99.8 percent of all individual trash bag breaking strengths.

8.3 Sample Size Determination

LO8-4 Determine the appropriate sample size when estimating a population mean.

In Section 8.1 we used a sample of 50 mileages to construct a 95 percent confidence interval for the midsize model's mean mileage μ. The size of this sample was not arbitrary—it was planned. To understand this, suppose that before the automaker selected the random sample of 50 mileages, it randomly selected the following sample of five mileages: 30.7, 31.9, 30.3, 32.0,

and 31.6. This sample has mean $\bar{x} = 31.3$. Assuming that the population of all mileages is normally distributed and that the population standard deviation σ is known to equal .8, it follows that a 95 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.025}\frac{\sigma}{\sqrt{n}}\right] = \left[31.3 \pm 1.96\frac{.8}{\sqrt{5}}\right]$$
$$= [31.3 \pm .701]$$
$$= [30.6, 32.0]$$

Although the sample mean $\bar{x} = 31.3$ is at least 31, the lower limit of the 95 percent confidence interval for μ is less than 31. Therefore, the midsize model's EPA mileage estimate would be 30 mpg, and the automaker would not receive its tax credit. One reason that the lower limit of this 95 percent interval is less than 31 is that the sample size of 5 is not large enough to make the interval's margin of error

$$z_{.025}\frac{\sigma}{\sqrt{n}} = 1.96\frac{.8}{\sqrt{5}} = .701$$

small enough. We can attempt to make the margin of error in the interval smaller by increasing the sample size. If we feel that the mean $\bar{x}$ of the larger sample will be at least 31.3 mpg (the mean of the small sample we have already taken), then the lower limit of a $100(1 - \alpha)$ percent confidence interval for μ will be at least 31 if the margin of error is .3 or less.

We will now explain how to find the size of the sample that will be needed to make the margin of error in a confidence interval for μ as small as we wish. In order to develop a formula for the needed sample size, we will initially assume that we know σ. Then, if the population is normally distributed or the sample size is large, the z-based $100(1 - \alpha)$ percent confidence interval for μ is $[\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})]$. To find the needed sample size, we set $z_{\alpha/2}(\sigma/\sqrt{n})$ equal to the desired margin of error and solve for n. Letting E denote the desired margin of error, we obtain

$$z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = E$$

Multiplying both sides of this equation by $\sqrt{n}$ and dividing both sides by E, we obtain

$$\sqrt{n} = \frac{z_{\alpha/2}\sigma}{E}$$

Squaring both sides of this result gives us the formula for n.

Determining the Sample Size for a Confidence Interval for μ: σ Known

A sample of size

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for μ equal to E. That is, this sample size makes us $100(1 - \alpha)$ percent confident that $\bar{x}$ is within E units of μ. If the calculated value of n is not a whole number, round this value up to the next whole number (so that the margin of error is at least as small as desired).

If we consider the formula for the sample size n, it intuitively follows that the value E is the farthest that the user is willing to allow $\bar{x}$ to be from μ at a given level of confidence, and the normal point $z_{\alpha/2}$ follows directly from the given level of confidence. Furthermore, because the population standard deviation σ is in the numerator of the formula for n, it follows that the more

variable that the individual population measurements are, the larger is the sample size needed to estimate μ with a specified accuracy.

In order to use this formula for n, we must either know σ (which is unlikely) or we must compute an estimate of σ. We first consider the case where we know σ. For example, suppose in the car mileage situation we wish to find the sample size that is needed to make the margin of error in a 95 percent confidence interval for μ equal to .3. Assuming that σ is known to equal .8, and using $z_{.025} = 1.96$, the appropriate sample size is

$$n = \left(\frac{z_{.025}\,\sigma}{E}\right)^2 = \left(\frac{1.96(.8)}{.3}\right)^2 = 27.32$$

Rounding up, we would employ a sample of size 28.

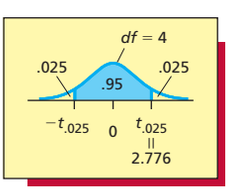
In most real situations, of course, we do not know the true value of σ. If σ is not known, we often estimate σ by using a preliminary sample. In this case we modify the above formula for n by replacing σ by the standard deviation s of the preliminary sample and by replacing $z_{\alpha/2}$ by $t_{\alpha/2}$. This approach usually gives a sample at least as large as we need. Thus, we obtain:

$$n = \left(\frac{t_{\alpha/2}\,s}{E}\right)^2$$

where the number of degrees of freedom for the $t_{\alpha/2}$ point is the size of the preliminary sample minus 1. Intuitively, using $t_{\alpha/2}$ compensates for the fact that the preliminary sample's value of s might underestimate σ, and, therefore, give a sample size that is too small.

EXAMPLE 8.4 The Car Mileage Case: Estimating Mean Mileage

Suppose that in the car mileage situation we wish to find the sample size that is needed to make the margin of error in a 95 percent confidence interval for μ equal to .3. Assuming we do not know σ, we regard the previously discussed sample of five mileages (see the bottom of page 317) as a preliminary sample. Therefore, we replace σ by the standard deviation of the preliminary sample, which can be calculated to be $s = .7583$, and we replace $z_{\alpha/2} = z_{.025} = 1.96$ by $t_{.025} = 2.776$, which is based on $n - 1 = 4$ degrees of freedom. We find that the appropriate sample size is



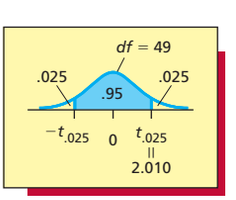
$$n = \left(\frac{t_{.025}\,s}{E}\right)^2 = \left(\frac{2.776(.7583)}{.3}\right)^2 = 49.24$$

Rounding up, we employ a sample of size 50.

When we make the margin of error in our 95 percent confidence interval for μ equal to .3, we can say we are 95 percent confident that the sample mean $\bar{x}$ is within .3 of μ. To understand this, suppose the true value of μ is 31.6. Recalling that the mean of the sample of 50 mileages is $\bar{x} = 31.56$, we see that this sample mean is within .3 of μ. Other samples of 50 mileages would give different sample means that would be different distances from μ. When we say that our sample of 50 mileages makes us 95 percent confident that $\bar{x}$ is within .3 of μ, we mean that **95 percent of all possible sample means based on 50 mileages are within .3 of μ** and 5 percent of such sample means are not.

EXAMPLE 8.5 The Car Mileage Case: Estimating Mean Mileage

To see that the sample of 50 mileages has actually produced a 95 percent confidence interval with a margin of error that is as small as we requested, recall that the 50 mileages have mean $\bar{x} = 31.56$ and standard deviation $s = .7977$. Therefore, the t-based 95 percent confidence interval for μ is



$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[31.56 \pm 2.010\frac{.7977}{\sqrt{50}}\right]$$
$$= [31.56 \pm .227]$$
$$= [31.33, 31.79]$$

where $t_{.025} = 2.010$ is based on $n - 1 = 50 - 1 = 49$ degrees of freedom—see Table A.4 (page 792). We see that the margin of error in this interval is .227, which is smaller than the .3 we asked for. Furthermore, as the automaker had hoped, the sample mean $\bar{x} = 31.56$ of the sample of 50 mileages turned out to be at least 31.3. Therefore, because the margin of error is less than .3, the lower limit of the 95 percent confidence interval is above 31 mpg, and the midsize model's EPA mileage estimate is 31 mpg. Because of this, the automaker will receive its tax credit.

Finally, sometimes we do not know σ and we do not have a preliminary sample that can be used to estimate σ. In this case it can be shown that, if we can make a reasonable guess of the range of the population being studied, then a conservatively large estimate of σ is this estimated range divided by 4. For example, if the automaker's design engineers feel that almost all of its midsize cars should get mileages within a range of 5 mpg, then a conservatively large estimate of σ is $5/4 = 1.25$ mpg. When employing such an estimate of σ, it is sufficient to use the z-based sample size formula $n = (z_{\alpha/2}\sigma/E)^2$, because a conservatively large estimate of σ will give us a conservatively large sample size.

Exercises for Section 8.3

CONCEPTS

connect

8.25 Explain what is meant by the margin of error for a confidence interval. What error are we talking about in the context of an interval for μ?

8.26 Explain exactly what we mean when we say that a sample of size n makes us 99 percent confident that $\bar{x}$ is within E units of μ.

8.27 Why do we often need to take a preliminary sample when determining the size of the sample needed to make the margin of error of a confidence interval equal to E?

METHODS AND APPLICATIONS

8.28 Consider a population having a standard deviation equal to 10. We wish to estimate the mean of this population.

- **a** How large a random sample is needed to construct a 95 percent confidence interval for the mean of this population with a margin of error equal to 1?
- **b** Suppose that we now take a random sample of the size we have determined in part *a*. If we obtain a sample mean equal to 295, calculate the 95 percent confidence interval for the population mean. What is the interval's margin of error?

8.29 Referring to Exercise 8.11*a* (page 309), assume that σ equals 33. How large a random sample of public owner-controlled companies is needed to make us:

- **a** 95 percent confident that $\bar{x}$, the sample mean audit delay, is within a margin of error of four days of μ, the population mean audit delay?
- **b** 99 percent confident that $\bar{x}$ is within a margin of error of four days of μ?

8.30 Referring to Exercise 8.12*b* (page 309), assume that σ equals .66. How large a sample of late replacement buyers is needed to make us:

- **a** 99 percent confident that $\bar{x}$, the sample mean number of dealers visited, is within a margin of error of .04 of μ, the population mean number of dealers visited?
- **b** 99.73 percent confident that $\bar{x}$ is within a margin of error of .05 of μ?

8.31 Referring to Exercise 8.21 (page 317), regard the sample of five trial runs for which $s = 19.65$ as a preliminary sample. Determine the number of trial runs of the chemical process needed to make us:

- **a** 95 percent confident that $\bar{x}$, the sample mean hourly yield, is within a margin of error of eight pounds of the population mean hourly yield μ when catalyst XA-100 is used.
- **b** 99 percent confident that $\bar{x}$ is within a margin of error of five pounds of μ. DS ChemYield

8.32 Referring to Exercise 8.20 (page 316), regard the sample of 10 sales figures for which $s = 32.866$ as a preliminary sample. How large a sample of sales figures is needed to make us 95 percent confident that $\bar{x}$, the sample mean sales dollars per square foot, is within a margin of error of \$10 of μ, the population mean sales dollars per square foot for all Whole Foods supermarkets. DS WholeFoods

8.33 **THE AIR SAFETY CASE** DS AlertTimes

Referring to Exercise 8.19 (page 316), regard the sample of 15 alert times for which $s = 1.026$ as a preliminary sample. Determine the sample size needed to make us 95 percent confident that $\bar{x}$, the sample mean alert time, is within a margin of error of .3 second of μ, the population mean alert time using the new display panel.

8.4 Confidence Intervals for a Population Proportion

LO8-5 Calculate and interpret a large sample confidence interval for a population proportion.

In Chapter 7, the soft cheese spread producer decided to replace its current spout with the new spout if p, the true proportion of all current purchasers who would stop buying the cheese spread if the new spout were used, is less than .10. Suppose that when 1,000 current purchasers are randomly selected and are asked to try the new spout, 63 say they would stop buying the spread if the new spout were used. The point estimate of the population proportion p is the sample proportion $\hat{p} = 63/1{,}000 = .063$. This sample proportion says we estimate that 6.3 percent of all current purchasers would stop buying the cheese spread if the new spout were used. Since $\hat{p}$ equals .063, we have some evidence that p is less than .10.

In order to see if there is strong evidence that p is less than .10, we can calculate a confidence interval for p. As explained in Chapter 7, if the sample size n is large, then the sampling distribution of the sample proportion $\hat{p}$ is approximately a normal distribution with mean $\mu_{\hat{p}} = p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$. By using the same logic we used in developing confidence intervals for μ, it follows that a $100(1-\alpha)$ percent confidence interval for p is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right]$$

Estimating $p(1-p)$ by $\hat{p}(1-\hat{p})$, it follows that a $100(1-\alpha)$ percent confidence interval for p can be calculated as summarized below.

A Large Sample Confidence Interval for a Population Proportion p

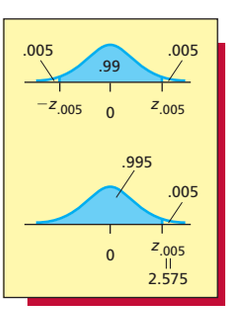
If the sample size n is large, **a $100(1-\alpha)$ percent confidence interval for the population proportion p** is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

Here n should be considered large if both $n\hat{p}$ and $n(1-\hat{p})$ are at least 5.[2]

EXAMPLE 8.6 The Cheese Spread Case: Improving Profitability

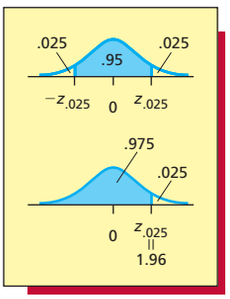
Suppose that the cheese spread producer wishes to calculate a 99 percent confidence interval for p, the population proportion of purchasers who would stop buying the cheese spread if the new spout were used. To determine whether the sample size $n = 1{,}000$ is large enough to enable us to use the confidence interval formula just given, recall that the point estimate of p is $\hat{p} = 63/1{,}000 = .063$. Therefore, because $n\hat{p} = 1{,}000(.063) = 63$ and $n(1-\hat{p}) = 1{,}000(.937) = 937$ are both greater than 5, we can use the confidence interval formula. It follows that the 99 percent confidence interval for p is



$$\left[\hat{p} \pm z_{.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = \left[.063 \pm 2.575\sqrt{\frac{(.063)(.937)}{1000}}\right]$$
$$= [.063 \pm .0198]$$
$$= [.0432, .0828]$$

This interval says that we are 99 percent confident that between 4.32 percent and 8.28 percent of all current purchasers would stop buying the cheese spread if the new spout were used. Moreover, because the upper limit of the 99 percent confidence interval is less than .10, we have very strong evidence that the true proportion p of all current purchasers who would stop buying the cheese spread is less than .10. Based on this result, it seems reasonable to use the new spout.

[2]Some statisticians suggest using the more conservative rule that both $n\hat{p}$ and $n(1-\hat{p})$ must be at least 10. Furthermore, because $\hat{p}(1-\hat{p})/(n-1)$ is an unbiased point estimate of $p(1-p)/n$, a more correct $100(1-\alpha)$ percent confidence interval for p is $[\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/(n-1)}]$. However, because n is large, there is little difference between intervals obtained by using this formula and those obtained by using the formula in the box above.



In order to compare the 99 percent confidence interval for p with a 95 percent confidence interval, we compute the 95 percent confidence interval as follows:

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = \left[.063 \pm 1.96\sqrt{\frac{(.063)(.937)}{1000}}\right]$$
$$= [.063 \pm .0151]$$
$$= [.0479, .0781]$$

Although the 99 percent confidence interval is somewhat longer than the 95 percent confidence interval, the fairly large sample size of $n = 1{,}000$ produces intervals that differ only slightly.

In the cheese spread example, a sample of 1,000 purchasers gives us a 99 percent confidence interval for p that has a margin of error of .0198 and a 95 percent confidence interval for p that has a margin of error of .0151. Both of these margins of errors are reasonably small. Generally, however, quite a large sample is needed in order to make the margin of error in a confidence interval for p reasonably small. The next two examples demonstrate that a sample size of 200, which most people would consider quite large, does not necessarily give a 95 percent confidence interval for p with a small margin of error.

EXAMPLE 8.7 The Phe-Mycin Case: Drug Side Effects

Antibiotics occasionally cause nausea as a side effect. Scientists working for a major drug company have developed a new antibiotic called Phe-Mycin. The company wishes to estimate p, the proportion of all patients who would experience nausea as a side effect when being treated with Phe-Mycin. Suppose that a sample of 200 patients is randomly selected. When these patients are treated with Phe-Mycin, 35 patients experience nausea. The point estimate of the population proportion p is the sample proportion $\hat{p} = 35/200 = .175$. This sample proportion says that we estimate that 17.5 percent of all patients would experience nausea as a side effect of taking Phe-Mycin. Furthermore, because $n\hat{p} = 200(.175) = 35$ and $n(1-\hat{p}) = 200(.825) = 165$ are both at least 5, we can use the previously given formula to calculate a confidence interval for p. Doing this, we find that a 95 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = \left[.175 \pm 1.96\sqrt{\frac{(.175)(.825)}{200}}\right]$$
$$= [.175 \pm .053]$$
$$= [.122, .228]$$

This interval says we are 95 percent confident that between 12.2 percent and 22.8 percent of all patients would experience nausea as a side effect of taking Phe-Mycin. Notice that the margin of error (.053) in this interval is rather large. Therefore, this interval is fairly long, and it does not provide a very precise estimate of p.

EXAMPLE 8.8 The Marketing Ethics Case: Confidentiality

In the book *Essentials of Marketing Research,* William R. Dillon, Thomas J. Madden, and Neil H. Firtle discuss a survey of marketing professionals, the results of which were originally published by Ishmael P. Akoah and Edward A. Riordan in the *Journal of Marketing Research*. In the study, randomly selected marketing researchers were presented with various scenarios involving ethical issues such as confidentiality, conflict of interest, and social acceptability. The marketing researchers were asked to indicate whether they approved or disapproved of the actions described

in each scenario. For instance, one scenario that involved the issue of confidentiality was described as follows:

> **Use of ultraviolet ink** A project director went to the marketing research director's office and requested permission to use an ultraviolet ink to precode a questionnaire for a mail survey. The project director pointed out that although the cover letter promised confidentiality, respondent identification was needed to permit adequate cross-tabulations of the data. The marketing research director gave approval.

Of the 205 marketing researchers who participated in the survey, 117 said they disapproved of the actions taken in the scenario. It follows that a point estimate of p, the proportion of all marketing researchers who disapprove of the actions taken in the scenario, is $\hat{p} = 117/205 = .5707$. Furthermore, because $n\hat{p} = 205(.5707) = 117$ and $n(1 - \hat{p}) = 205(.4293) = 88$ are both at least 5, a 95 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = \left[.5707 \pm 1.96\sqrt{\frac{(.5707)(.4293)}{205}}\right]$$
$$= [.5707 \pm .0678]$$
$$= [.5029, .6385]$$

This interval says we are 95 percent confident that between 50.29 percent and 63.85 percent of all marketing researchers disapprove of the actions taken in the ultraviolet ink scenario. Notice that because the margin of error (.0678) in this interval is rather large, this interval does not provide a very precise estimate of p. Below we show the MINITAB output of this interval.

CI for One Proportion

X	N	Sample p	95% CI
117	205	0.570732	(0.502975, 0.638488)

LO8-6 Determine the appropriate sample size when estimating a population proportion.

In order to find the size of the sample needed to estimate a population proportion, we consider the theoretically correct interval

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right]$$

To obtain the sample size needed to make the margin of error in this interval equal to E, we set

$$z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} = E$$

and solve for n. When we do this, we get the following result:

Determining the Sample Size for a Confidence Interval for p

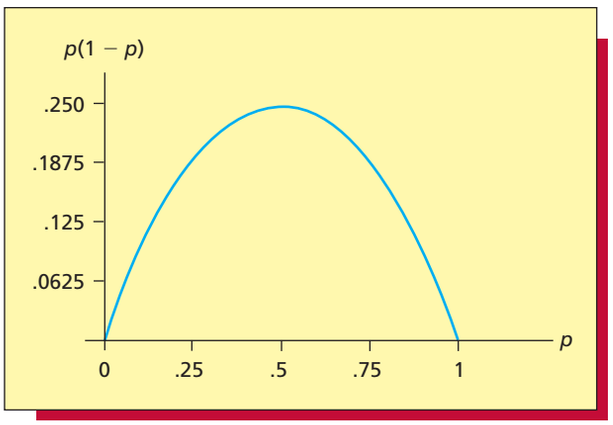
A sample of size

$$n = p(1-p)\left(\frac{z_{\alpha/2}}{E}\right)^2$$

makes the margin of error in a $100(1-\alpha)$ percent confidence interval for p equal to E. That is, this sample size makes us $100(1-\alpha)$ percent confident that $\hat{p}$ is within E units of p. If the calculated value of n is not a whole number, round this value up to the next whole number.

Looking at this formula, we see that the larger $p(1-p)$ is, the larger n will be. To make sure n is large enough, consider Figure 8.16 on the next page, which is a graph of $p(1-p)$ versus p. This figure shows that $p(1-p)$ equals .25 when p equals .5. Furthermore, $p(1-p)$ is never larger than .25. Therefore, if the true value of p could be near .5, we should set $p(1-p)$ equal to .25. This will ensure that n is as large as needed to make the margin of error as small as desired. For

FIGURE 8.16 The Graph of $p(1 - p)$ versus p



example, suppose we wish to estimate the proportion p of all registered voters who currently favor a particular candidate for president of the United States. If this candidate is the nominee of a major political party, or if the candidate enjoys broad popularity for some other reason, then p could be near .5. Furthermore, suppose we wish to make the margin of error in a 95 percent confidence interval for p equal to .02. If the sample to be taken is random, it should consist of

$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.96}{.02}\right)^2 = 2{,}401$$

registered voters. In reality, a list of all registered voters in the United States is not available to polling organizations. Therefore, it is not feasible to take a (technically correct) random sample of registered voters. For this reason, polling organizations actually employ other (more complicated) kinds of samples. We have explained some of the basic ideas behind these more complex samples in optional Section 7.4. For now, we consider the samples taken by polling organizations to be approximately random. Suppose, then, that when the sample of voters is actually taken, the proportion $\hat{p}$ of sampled voters who favor the candidate turns out to be greater than .52. It follows, because the sample is large enough to make the margin of error in a 95 percent confidence interval for p equal to .02, that the lower limit of such an interval is greater than .50. This says we have strong evidence that a majority of all registered voters favor the candidate. For instance, if the sample proportion $\hat{p}$ equals .53, we are 95 percent confident that the proportion of all registered voters who favor the candidate is between .51 and .55.

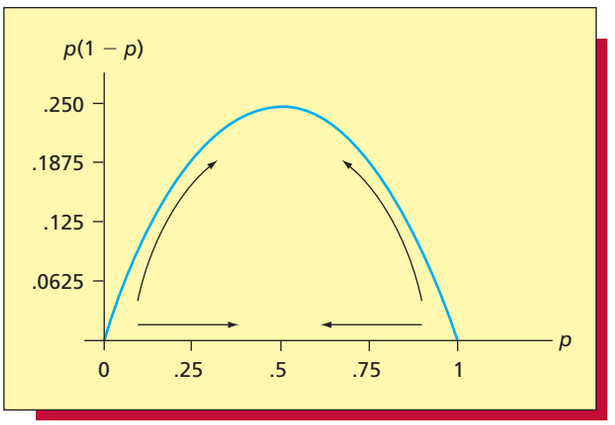
Major polling organizations conduct public opinion polls concerning many kinds of issues. While making the margin of error in a 95 percent confidence interval for p equal to .02 requires a sample size of 2,401, making the margin of error in such an interval equal to .03 requires a sample size of only

$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.96}{.03}\right)^2 = 1{,}067.1$$

or 1,068 (rounding up). Of course, these calculations assume that the proportion p being estimated could be near .5. However, for any value of p, increasing the margin of error from .02 to .03 substantially decreases the needed sample size and thus saves considerable time and money. For this reason, although the most accurate public opinion polls use a margin of error of .02, the vast majority of public opinion polls use a margin of error of .03 or larger.

When the news media report the results of a public opinion poll, they express the margin of error in a 95 percent confidence interval for p *in percentage points.* For instance, if the margin of error is .03, the media would say the poll's margin of error is 3 percentage points. The media seldom report the level of confidence, but almost all polling results are based on 95 percent confidence. Sometimes the media make a vague reference to the level of confidence. For instance, if the margin of error is 3 percentage points, the media might say that "the sample result will be within 3 percentage points of the population value in 19 out of 20 samples." Here

FIGURE 8.17 As p Gets Closer to .5, $p(1 - p)$ Increases



the "19 out of 20 samples" is a reference to the level of confidence, which is $100(19/20) = 100(.95) = 95$ percent.

As an example, suppose a news report says a recent poll finds that 34 percent of the public favors military intervention in an international crisis, and suppose the poll's margin of error is reported to be 3 percentage points. This means the sample taken is large enough to make us 95 percent confident that the sample proportion $\hat{p} = .34$ is within .03 (that is, 3 percentage points) of the true proportion p of the entire public that favors military intervention. That is, we are 95 percent confident that p is between .31 and .37.

If the population proportion we are estimating is substantially different from .5, setting p equal to .5 will give a sample size that is much larger than is needed. In this case, we should use our intuition or previous sample information (along with Figure 8.17) to determine the largest reasonable value for $p(1 - p)$. Figure 8.17 implies that as p gets closer to .5, $p(1 - p)$ increases. It follows that $p(1 - p)$ is maximized by the reasonable value of p that is closest to .5. Therefore, **when we are estimating a proportion that is substantially different from .5, we use the reasonable value of p that is closest to .5 to calculate the sample size needed to obtain a specified margin of error.**

EXAMPLE 8.9 The Phe-Mycin Case: Drug Side Effects

Again consider estimating the proportion of all patients who would experience nausea as a side effect of taking the new antibiotic Phe-Mycin. Suppose the drug company wishes to find the size of the random sample that is needed in order to obtain a 2 percent margin of error with 95 percent confidence. In Example 8.7 we employed a sample of 200 patients to compute a 95 percent confidence interval for p. This interval, which is [.122, .228], makes us very confident that p is between .122 and .228. Because .228 is the reasonable value of p that is closest to .5, the largest reasonable value of $p(1 - p)$ is $.228(1 - .228) = .1760$, and thus the drug company should take a sample of

$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{E}\right)^2 = .1760\left(\frac{1.96}{.02}\right)^2 = 1{,}691 \text{ (rounded up)}$$

patients.

Finally, as a last example of choosing p for sample size calculations, suppose that experience indicates that a population proportion p is at least .75. Then, .75 is the reasonable value of p that is closest to .5, and we would use the largest reasonable value of $p(1 - p)$, which is $.75(1 - .75) = .1875$.

Exercises for Section 8.4

CONCEPTS

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8.34 **a** What does a population proportion tell us about the population?
b Explain the difference between p and $\hat{p}$.
c What is meant when a public opinion poll's *margin of error* is 3 percent?

8.35 Suppose we are using the sample size formula in the box on page 323 to find the sample size needed to make the margin of error in a confidence interval for p equal to E. In each of the following situations, explain what value of p would be used in the formula for finding n:
a We have no idea what value p is—it could be any value between 0 and 1.
b Past experience tells us that p is no more than .3.
c Past experience tells us that p is at least .8.

METHODS AND APPLICATIONS

8.36 In each of the following cases, determine whether the sample size n is large enough to use the large sample formula presented in the box on page 321 to compute a confidence interval for p.
a $\hat{p} = .1, \quad n = 30$
b $\hat{p} = .1, \quad n = 100$
c $\hat{p} = .5, \quad n = 50$
d $\hat{p} = .8, \quad n = 400$
e $\hat{p} = .9, \quad n = 30$
f $\hat{p} = .99, \quad n = 200$

8.37 In each of the following cases, compute 95 percent, 98 percent, and 99 percent confidence intervals for the population proportion p.
a $\hat{p} = .4$ and $n = 100$
b $\hat{p} = .1$ and $n = 300$
c $\hat{p} = .9$ and $n = 100$
d $\hat{p} = .6$ and $n = 50$

8.38 In a news story distributed by the *Washington Post,* Lew Sichelman reports that a substantial fraction of mortgage loans that go into default within the first year of the mortgage were approved on the basis of falsified applications. For instance, loan applicants often exaggerate their income or fail to declare debts. Suppose that a random sample of 1,000 mortgage loans that were defaulted within the first year reveals that 410 of these loans were approved on the basis of falsified applications.
a Find a point estimate of and a 95 percent confidence interval for p, the proportion of all first-year defaults that are approved on the basis of falsified applications.
b Based on your interval, what is a reasonable estimate of the minimum percentage of all first-year defaults that are approved on the basis of falsified applications?

8.39 Suppose that 60 percent of 1000 randomly selected U.S. adults say that they take part in some form of daily activity to keep physically fit. Based on this finding, find a 95 percent confidence interval for the proportion of all U.S. adults who would say they take part in some form of daily activity to keep physically fit. Based on this interval, is it reasonable to conclude that more than 50 percent of all U.S. adults would say they take part in some form of daily activity to keep physically fit?

8.40 In an article in the *Journal of Advertising,* Weinberger and Spotts compare the use of humor in television ads in the United States and the United Kingdom. They found that a substantially greater percentage of U.K. ads use humor.
a Suppose that a random sample of 400 television ads in the United Kingdom reveals that 142 of these ads use humor. Find a point estimate of and a 95 percent confidence interval for the proportion of all U.K. television ads that use humor.
b Suppose a random sample of 500 television ads in the United States reveals that 122 of these ads use humor. Find a point estimate of and a 95 percent confidence interval for the proportion of all U.S. television ads that use humor.
c Do the confidence intervals you computed in parts *a* and *b* suggest that a greater percentage of U.K. ads use humor? Explain. How might an ad agency use this information?

8.41 THE MARKETING ETHICS CASE: CONFLICT OF INTEREST

Consider the marketing ethics case described in Example 8.8. One of the scenarios presented to the 205 marketing researchers is as follows:

> A marketing testing firm to which X company gives most of its business recently went public. The marketing research director of X company had been looking for a good investment and proceeded to buy some $20,000 of their stock. The firm continues as X company's leading supplier for testing.

Of the 205 marketing researchers who participated in the ethics survey, 111 said that they disapproved of the actions taken in the scenario. Use this sample result to show that the 95 percent

confidence interval for the proportion of all marketing researchers who disapprove of the actions taken in the conflict of interest scenario is as given in the MINITAB output below. Interpret this interval.

CI for One Proportion

X	N	Sample p	95% CI
111	205	0.541463	(0.473254, 0.609673)

8.42 On the basis of the confidence interval given in Exercise 8.41, is there convincing evidence that a majority of all marketing researchers disapprove of the actions taken in the conflict of interest scenario? Explain.

8.43 In an article in *CA Magazine,* Neil Fitzgerald surveyed Scottish business customers concerning their satisfaction with aspects of their banking relationships. Fitzgerald reports that, in 418 telephone interviews conducted by George Street Research, 67 percent of the respondents gave their banks a high rating for overall satisfaction.

a Assuming that the sample is randomly selected, calculate a 99 percent confidence interval for the proportion of all Scottish business customers who give their banks a high rating for overall satisfaction.

b Based on this interval, can we be 99 percent confident that more than 60 percent of all Scottish business customers give their banks a high rating for overall satisfaction?

8.44 The manufacturer of the ColorSmart-5000 television set claims 95 percent of its sets last at least five years without needing a single repair. In order to test this claim, a consumer group randomly selects 400 consumers who have owned a ColorSmart-5000 television set for five years. Of these 400 consumers, 316 say their ColorSmart-5000 television sets did not need a repair, whereas 84 say their ColorSmart-5000 television sets did need at least one repair.

a Find a 99 percent confidence interval for the proportion of all ColorSmart-5000 television sets that have lasted at least five years without needing a single repair.

b Does this confidence interval provide strong evidence that the percentage of all ColorSmart-5000 television sets that last at least five years without a single repair is less than the 95 percent claimed by the manufacturer? Explain.

8.45 *Consumer Reports* (January 2005) indicates that profit margins on extended warranties are much greater than on the purchase of most products.[3] In this exercise we consider a major electronics retailer that wishes to increase the proportion of customers who buy extended warranties on digital cameras. Historically, 20 percent of digital camera customers have purchased the retailer's extended warranty. To increase this percentage, the retailer has decided to offer a new warranty that is less expensive and more comprehensive. Suppose that three months after starting to offer the new warranty, a random sample of 500 customer sales invoices shows that 152 out of 500 digital camera customers purchased the new warranty. Find a 95 percent confidence interval for the proportion of all digital camera customers who have purchased the new warranty. Are we 95 percent confident that this proportion exceeds .20?

8.46 Consider Exercise 8.39 and suppose we wish to find the sample size n needed in order to be 95 percent confident that $\hat{p}$, the sample proportion of respondents who said they took part in some sort of daily activity to keep physically fit, is within a margin of error of .02 of p, the proportion of all U.S. adults who say that they take part in such activity. In order to find an appropriate value for $p(1 - p)$, note that the 95 percent confidence interval for p that you calculated in Exercise 8.39 was [.57, .63]. This indicates that the reasonable value for p that is closest to .5 is .57, and thus the largest reasonable value for $p(1 - p)$ is $.57(1 - .57) = .2451$. Calculate the required sample size n.

8.47 Referring to Exercise 8.44, determine the sample size needed in order to be 99 percent confident that $\hat{p}$, the sample proportion of ColorSmart-5000 television sets that last at least five years without a single repair, is within a margin of error of .03 of p, the proportion of all sets that last at least five years without a single repair.

8.48 Suppose we conduct a poll to estimate the proportion of voters who favor a major presidential candidate. Assuming that 50 percent of the electorate could be in favor of the candidate, determine the sample size needed so that we are 95 percent confident that $\hat{p}$, the sample proportion of voters who favor the candidate, is within a margin of error of .01 of p, the proportion of all voters who are in favor of the candidate.

[3] *Consumer Reports*, January 2005, page 51.

LO8-7 Find and interpret confidence intervals for parameters of finite populations (Optional).

8.5 Confidence Intervals for Parameters of Finite Populations (Optional)

Random sampling Companies in financial trouble have sometimes falsified their accounts receivable invoices in order to mislead stockholders. For this reason, independent auditors are often asked to estimate a company's true total sales for a given period. To illustrate this, consider a company that sells home theaters. The company accumulated 2,418 sales invoices last year. The total of the sales amounts listed on these invoices (that is, the total sales claimed by the company) is \$5,127,492.17. In order to estimate the true total sales for last year, an independent auditor randomly selects 242 of the invoices without replacement and determines the actual sales amounts by contacting the purchasers. The mean and the standard deviation of the actual sales amounts for these invoices are $\bar{x} = \$1{,}843.93$ and $s = \$516.42$. To use this sample information to estimate the company's true total sales, note that the mean μ of a finite population is the **population total** τ, which is the sum of the values of all of the population measurements, divided by the number, N, of population measurements. That is, we have $\mu = \tau/N$, which implies that $\tau = N\mu$. Because a point estimate of the population mean μ is the sample mean $\bar{x}$, it follows that *a point estimate of the population total* τ is $N\bar{x}$. In the context of estimating the true total sales τ for the home theater company's population of $N = 2{,}418$ sales invoices, the sample mean invoice amount is $\bar{x} = \$1{,}843.93$, and thus the point estimate of τ is $N\bar{x} = 2{,}418(\$1{,}843.93) = \$4{,}458{,}622.74$. This point estimate is considerably lower than the claimed total sales of \$5,127,492.17. However, we cannot expect the point estimate of τ to exactly equal the true total sales, so we need to calculate a confidence interval for τ before drawing any unwarranted conclusions.

In general, consider randomly selecting without replacement a large sample of n measurements from a finite population consisting of N measurements and having mean μ and standard deviation σ. It can then be shown that the sampling distribution of the sample mean $\bar{x}$ is approximately a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = (\sigma/\sqrt{n})\sqrt{(N-n)/(N-1)}$. Estimating σ by the sample standard deviation s, it can also be shown that the appropriate point estimate of $\sigma_{\bar{x}}$ is $(s/\sqrt{n})\sqrt{(N-n)/N}$, which is used in the following result.

Confidence Intervals for the Population Mean and Population Total for a Finite Population

Suppose we randomly select a sample of n measurements **without replacement from a finite population of N measurements.** Then, if n is large (say, at least 30)

1 **A 100(1 — α) percent confidence interval for the population mean μ is**

$$\left[\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N}}\right]$$

2 **A 100(1 — α) percent confidence interval for the population total τ is** found by multiplying the lower and upper limits of the 100(1 — α) percent confidence interval for μ by N.

The quantity $\sqrt{(N-n)/N}$ in the confidence intervals for μ and τ is called the **finite population correction** and is always less than 1. If the population size N is much larger than the sample size n, then the finite population correction is only slightly less than 1. For example, if we randomly select (without replacement) a sample of 1,000 from a population of 1 million, then the finite population correction is $\sqrt{(1{,}000{,}000 - 1{,}000)/1{,}000{,}000} = .9995$. In such a case, the finite population correction is not enough less than 1 to meaningfully shorten the confidence intervals for μ and τ and thus can be set equal to 1. However, **if the population size N is not much larger than the sample size n (say, if n is more than 5 percent of N), then the finite population correction is substantially less than 1 and should be included** in the confidence interval calculations.

EXAMPLE 8.10 The Home Theater Case: Auditing Total Sales

Recall that when the independent auditor randomly selects a sample of $n = 242$ invoices, the mean and standard deviation of the actual sales amounts for these invoices are $\bar{x} = 1{,}843.93$ and $s = 516.42$. Here the sample size $n = 242$ is $(242/2{,}418) \times 100 = 10.008$ percent of the population size $N = 2{,}418$. Because n is more than 5 percent of N, we should include the finite population correction in our confidence interval calculations. It follows that a 95 percent confidence interval for the true mean sales amount μ per invoice is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N}}\right] = \left[1{,}843.93 \pm 1.96\frac{516.42}{\sqrt{242}}\sqrt{\frac{2{,}418-242}{2{,}418}}\right]$$
$$= [1{,}843.93 \pm 61.723812]$$

or [\$1,782.21, \$1,905.65]. Moreover, multiplying the lower and upper limits of this interval by $N = 2{,}418$, we find that a 95 percent confidence interval for the true total sales τ is [1,782.21(2,418), 1,905.65(2,418)], or [\$4,309,383.80, \$4,607,861.70]. Because the upper limit of this interval is more than \$500,000 below the claimed total sales amount of \$5,127,492.17, we have strong evidence that the claimed total sales amount is overstated.

We sometimes estimate the total number, τ, of population units that fall into a particular category. For instance, the auditor of Example 8.10 might wish to estimate the total number of the 2,418 invoices having incorrect sales amounts. Here the proportion, p, of the population units that fall into a particular category is the total number, τ, of population units that fall into the category divided by the number, N, of population units. That is, $p = \tau/N$, which implies that $\tau = Np$. Therefore, because a point estimate of the population proportion p is the sample proportion $\hat{p}$, a point estimate of the population total τ is $N\hat{p}$. For example, suppose that 34 of the 242 sampled invoices have incorrect sales amounts. Because the sample proportion is $\hat{p} = 34/242 = .1405$, a point estimate of the total number of the 2,418 invoices that have incorrect sales amounts is $N\hat{p} = 2{,}418(.1405) = 339.729$.

Confidence Intervals for the Proportion of and Total Number of Units in a Category When Sampling a Finite Population

Suppose that we randomly select a sample of n units **without replacement from a finite population of N units**. Then, if n is large

1 **A 100(1 — α) percent confidence interval for the population proportion p** is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}\left(\frac{N-n}{N}\right)}\right]$$

2 **A 100(1 — α) percent confidence interval for the population total τ is** found by multiplying the lower and upper limits of the 100(1 — α) percent confidence interval for p by N.

EXAMPLE 8.11 The Home Theater Case: Auditing Sales Invoices

Recall that we found that 34 of the 242 sampled invoices have incorrect sales amounts. Because $\hat{p} = 34/242 = .1405$, a 95 percent confidence interval for the true proportion of the 2,418 invoices that have incorrect sales amounts is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}\left(\frac{N-n}{N}\right)}\right] = \left[.1405 \pm 1.96\sqrt{\frac{(.1405)(.8595)}{242}\left(\frac{2{,}418-242}{2{,}418}\right)}\right]$$
$$= [.1405 \pm .0416]$$

or [.0989, .1821]. Moreover, multiplying the lower and upper limits of this interval by $N = 2{,}418$, we find that a 95 percent confidence interval for the true total number, τ, of the 2,418 sales invoices that have incorrect sales amounts is [.0989(2,418), .1821(2,418)], or [239.14, 440.32]. Therefore, we are 95 percent confident that between (roughly) 239 and 440 of the 2,418 invoices have incorrect sales amounts.

Stratified random sampling Recall from Section 7.4 (page 287) that in order to select a *stratified random sample*, we partition the population into nonoverlapping subpopulations called *strata*. Then, we select a random sample from each *stratum*. Also, recall that we have considered a department store chain that proposes to open a new store in a location that would serve a geographical region consisting of three strata: an industrial city, a suburban community, and a rural area. In order to estimate (1) the mean household income in this region and (2) the total number of households in the region that would be likely to patronize the new store, the department store chain randomly selects without replacement 1 percent of the households from each stratum. Each household is asked to (1) report its total household income for the previous year, and (2) indicate whether at least one member of the household would be likely to shop at the new store. Table 8.4 summarizes the size of each stratum (N_i): the size of the sample selected from each stratum (n_i); the mean, variance, and standard deviation of the household incomes sampled from each stratum ($\bar{x}_i$, s_i^2, and s_i); and the proportion of sampled households in each stratum indicating that at least one member of the household would be likely to shop at the new store ($\hat{p}_i$). We now consider the following results.

Estimation in Stratified Random Sampling

Assume that a population is partitioned into k strata, and assume that we randomly select without replacement a sample of n_i measurements from the N_i measurements in the ith stratum. If $\bar{x}_i$, s_i^2, and $\hat{p}_i$ denote the sample mean, sample variance, and sample proportion for the sample selected from the ith stratum, then it can be shown that:

1 **A point estimate of the total, τ, of the values of all of the measurements in the entire population is $\hat{\tau} = N_1\bar{x}_1 + N_2\bar{x}_2 + \cdots + N_k\bar{x}_k$.** Moreover, if each n_i is large (say, at least 30), **a $100(1-\alpha)$ percent confidence interval for τ** is

$$\left[\hat{\tau} \pm z_{\alpha/2}\sqrt{\sum_{i=1}^{k} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\frac{s_i^2}{n_i}}\right]$$

2 Noting that the total population size is $N = N_1 + N_2 + \cdots + N_k$, **a point estimate of the population mean μ is $\hat{\mu} = \hat{\tau}/N$, and a $100(1-\alpha)$ percent confidence interval for μ** is found by dividing the lower and upper limits of the $100(1-\alpha)$ percent confidence interval for τ by N.

3 **A point estimate of the total, τ, of the units in the entire population that fall into a category of interest is $\hat{\tau} = N_1\hat{p}_1 + N_2\hat{p}_2 + \cdots + N_k\hat{p}_k$.** Moreover, if each n_i is large, **a $100(1-\alpha)$ percent confidence interval for τ** is

$$\left[\hat{\tau} \pm z_{\alpha/2}\sqrt{\sum_{i=1}^{k} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\frac{\hat{p}_i(1-\hat{p}_i)}{n_i}}\right]$$

4 **A point estimate of the population proportion p is $\hat{p} = \hat{\tau}/N$,** and **a $100(1-\alpha)$ percent confidence interval for p** is found by dividing the lower and upper limits of the $100(1-\alpha)$ percent confidence interval for τ by N.

TABLE 8.4 A Stratified Random Sample in the Department Store Chain Example

Stratum	N_i	n_i	$\bar{x}_i$	s_i^2	s_i	$\hat{p}_i$
1. Industrial city	12,172	122	\$47,287	9,294,621	3048.7	66/122 = .5410
2. Suburb	6,914	69	68,719	25,486,211	5048.4	52/69 = .7536
3. Rural area	3,281	33	56,218	17,186,592	4145.7	22/33 = .6667
	$N = 22{,}367$					

EXAMPLE 8.12 The Department Store Case: Studying Market Demographics

The department store chain has concluded that it should open the new store in the geographical region if (1) it is 95 percent confident that the mean income per household is at least \$50,000 and (2) it is 95 percent confident that the total number of households for which at least one household member would be likely to shop at the new store is at least 10,000. Using the information in Table 8.4, a point estimate of the total income, τ, of all households in the geographical region is $\hat{\tau} = N_1\bar{x}_1 + N_2\bar{x}_2 + N_3\bar{x}_3 = 12{,}172(47{,}287) + 6{,}914(68{,}719) + 3{,}281(56{,}218) = 1{,}235{,}150{,}000$ (dollars). To compute a 95 percent confidence interval for τ, we first compute the margin of error in this interval, which is

$$z_{.025}\sqrt{\sum_{i=1}^{k} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\frac{s_i^2}{n_i}}$$

$$= 1.96\sqrt{(12{,}172)^2\left(\frac{9{,}294{,}621}{122}\right) + (6{,}914)^2\left(\frac{25{,}486{,}211}{69}\right) + (3{,}281)^2\left(\frac{17{,}186{,}592}{33}\right)}$$

$$= 11{,}520{,}869.50$$

Here, we have set each $(N_i - n_i)/N_i$ equal to 1, because each n_i is only 1 percent of N_i. It follows that the 95 percent confidence interval for τ is [1,235,150,000 ± 11,520,869.50], or [\$1,223,629,130.50, \$1,246,670,869.50]. Dividing the point estimate of τ, $\hat{\tau} = \$1{,}235{,}150{,}000$, by the population size $N = 22{,}367$ (see Table 8.4), we find that the point estimate of μ, the mean income per household, is $\hat{\mu} = \$55{,}222$. Dividing the lower and upper limits of the 95 percent confidence interval for τ by $N = 22{,}367$, we find that the 95 percent confidence interval for μ is [\$54,707, \$55,737]. This interval makes the department store chain 95 percent confident that the mean income per household is at least \$50,000. Therefore, the first condition for opening the new store is met.

Now considering the second condition for opening the new store, note that the point estimate of the true total number, τ, of households in the geographical region for which at least one household member would be likely to shop at the new store is $\hat{\tau} = N_1\hat{p}_1 + N_2\hat{p}_2 + N_3\hat{p}_3 = 12{,}172(.5410) + 6{,}914(.7536) + 3{,}281(.6667) = 13{,}983$. In addition, the margin of error in the 95 percent confidence interval for τ is

$$z_{.025}\sqrt{\sum_{i=1}^{k} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\frac{\hat{p}_i(1 - \hat{p}_i)}{n_i}}$$

$$= 1.96\sqrt{(12{,}172)^2\frac{(.5410)(.4590)}{122} + (6{,}914)^2\frac{(.7536)(.2464)}{69} + (3{,}281)^2\frac{(.6667)(.3333)}{33}} = 1{,}399$$

where we again have set each $(N_i - n_i)/N_i$ equal to 1. It follows that the 95 percent confidence interval for τ is [13,983 ± 1399], or [12,584, 15,382]. This interval makes the department store chain 95 percent confident that the true total number of households in the geographical region for which at least one household member would be likely to shop at the new store is between 12,584 households and 15,382 households. Because the department store chain is 95 percent confident that this total is at least 10,000, the second condition for opening the new store is met, and the new store will be opened.

To conclude this example, we will find a point estimate of and a 95 percent confidence interval for the true proportion, p, of households in the geographical region for which at least one household member would be likely to shop at the new store. Dividing the previously calculated point estimate of τ, $\hat{\tau} = 13{,}983$, by the total number of households in the geographical region, $N = 22{,}367$, we find that the point estimate of p is .6252. Dividing the lower and upper limits of the previously calculated 95 percent confidence interval for τ, [12,584, 15,832], by $N = 22{,}367$, we find that the 95 percent confidence interval for p is [.5626, .6877].

Exercises for Section 8.5

CONCEPTS

8.49 Define a population total. Give an example of a population total that will interest you in your career when you graduate from college.

8.50 Explain why the finite population correction $\sqrt{(N - n)/N}$ is unnecessary when the population is at least 20 times as large as the sample. Give an example using numbers.

METHODS AND APPLICATIONS

8.51 A retailer that sells audio and video equipment accumulated 10,451 sales invoices during the previous year. The total of the sales amounts listed on these invoices (that is, the total sales claimed by the company) is \$6,384,675. In order to estimate the true total sales for last year, an independent auditor randomly selects 350 of the invoices and determines the actual sales amounts by contacting the purchasers. The mean and the standard deviation of the 350 sampled sales amounts are $\bar{x} = \$532$ and $s = \$168$.

a Find a 95 percent confidence interval for μ, the true mean sales amount per invoice on the 10,451 invoices.

b Find a point estimate of and a 95 percent confidence interval for τ, the true total sales for the previous year.

c What does this interval say about the company's claim that the true total sales were \$6,384,675? Explain.

8.52 A personnel manager is estimating the total number of person-days lost to unexcused absences by hourly workers in the last year. In a random sample of 50 employees drawn without replacement from the 687 hourly workers at the company, records show that the 50 sampled workers had an average of $\bar{x} = 4.3$ days of unexcused absences over the past year with a standard deviation of $s = 1.26$.

a Find a point estimate of and a 95 percent confidence interval for the total number of unexcused absences by hourly workers in the last year.

b Can the personnel manager be 95 percent confident that more than 2,500 person-days were lost to unexcused absences last year? Can the manager be 95 percent confident that more than 3,000 person-days were lost to unexcused absences last year? Explain.

8.53 A company's manager is considering simplification of a travel voucher form. In order to assess the costs associated with erroneous travel vouchers, the manager must estimate the total number of such vouchers that were filled out incorrectly in the last month. In a random sample of 100 vouchers drawn without replacement from the 1,323 travel vouchers submitted in the last month, 31 vouchers were filled out incorrectly.

a Find a point estimate of and a 95 percent confidence interval for the true proportion of travel vouchers that were filled out incorrectly in the last month.

b Find a point estimate of and a 95 percent confidence interval for the total number of travel vouchers that were filled out incorrectly in the last month.

c If it costs the company \$10 to correct an erroneous travel voucher, find a reasonable estimate of the minimum cost of correcting all of last month's erroneous travel vouchers. Would it be worthwhile to spend \$5,000 to design a simplified travel voucher that could be used for at least a year?

8.54 SAMPLE SIZES WHEN RANDOM SAMPLING FINITE POPULATIONS

a *Estimating μ and τ*

Consider randomly selecting a sample of n measurements without replacement from a finite population consisting of N measurements and having variance σ^2. Also consider the sample size given by the formula $n = N\sigma^2/[(N - 1)D + \sigma^2]$. It can be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for μ equal to E if we set D equal to $(E/z_{\alpha/2})^2$. It can also be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for τ equal to E if we set D equal to $[E/(z_{\alpha/2}N)]^2$. Now consider Exercise 8.52. Using $s^2 = (1.26)^2 = 1.5876$ as an estimate of σ^2, determine the sample size that makes the margin of error in a 95 percent confidence interval for the *total number* of person-days lost to unexcused absences last year equal to 100 days.

b *Estimating p and τ*

Consider randomly selecting a sample of n units without replacement from a finite population consisting of N units and having a proportion p of these units fall into a particular category.

Also, consider the sample size given by the formula $n = Np(1 - p)/[(N - 1)D + p(1 - p)]$. It can be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for p equal to E if we set D equal to $(E/z_{\alpha/2})^2$. It can also be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for τ equal to E if we set D equal to $[E/(z_{\alpha/2}N)]^2$. Now consider Exercise 8.53. Using $\hat{p} = .31$ as an estimate of p, determine the sample size that makes the margin of error in a 95 percent confidence interval for the *proportion* of the 1,323 vouchers that were filled out incorrectly equal to .04.

8.55 The accounts of an investment firm have been classified into 143 large accounts, 4,780 medium-sized accounts, and 20,973 small accounts. The investment firm provides an auditor with each account's *book value*. This is the amount of money that the firm claims is in the account. The auditor determines an account's *audit value* by contacting the owner of the account. This is the amount of money that is actually in the account.

The auditor wishes to detect discrepancies between the stated book values and the audit values. Because it is time consuming and expensive to obtain the audit values, a stratified random sample of accounts is selected, and the difference between the book value and the audit value is determined for each sampled account. The following results are obtained.

Stratum	Number of Accounts in Stratum	Sample Size	Sample Mean Difference	Sample Standard Deviation	Sample Proportion of Accounts Overstated
Large	143	72	$10,661	$5,056	.83
Medium-sized	4,780	239	1,213	671	.94
Small	20,973	210	19	159	.37

a Find point estimates of and 95 percent confidence intervals for the true total, τ, and the true mean, μ, of the differences between the book values and the audit values for the population of all of the investment firm's accounts.

b Find point estimates of and 95 percent confidence intervals for the true total number, τ, and the true proportion, p, of overstated accounts among all of the investment firm's accounts.

c What do you conclude? Why was stratified random sampling important in this situation?

8.56 SAMPLE SIZES IN STRATIFIED RANDOM SAMPLING

Although the department store chain in Example 8.12 has randomly selected 1 percent of the homes in each stratum, there is a more precise way to determine how to sample the strata. The first step is to determine the **optimal fraction,** f_i, of the total sample size, n, that will be **allocated** to the ith stratum. This optimal fraction can be proven to give the shortest confidence intervals for a fixed sample size and can be proven to equal

$$f_i = \frac{N_i\sigma_i/\sqrt{c_i}}{N_1\sigma_1/\sqrt{c_1} + N_2\sigma_2/\sqrt{c_2} + \cdots + N_k\sigma_k/\sqrt{c_k}}$$

Here, c_i is the cost of sampling the ith stratum, and σ_i is the true standard deviation of the measurements in the ith stratum. If the sampling costs are equal (as they might be when conducting telephone interviews), $f_i = N_i\sigma_i/(N_1\sigma_1 + N_2\sigma_2 + \cdots + N_k\sigma_k)$. In the department store example, we will assume that the sampling costs are equal, and we will estimate σ_1, σ_2, and σ_3 by s_1, s_2, and s_3. We can then estimate $N_1\sigma_1$, $N_2\sigma_2$, and $N_3\sigma_3$ to be $N_1s_1 = 37{,}108{,}776$, $N_2s_2 = 34{,}904{,}638$, and $N_3s_3 = 13{,}602{,}042$. Here, we have used the N_i and s_i values in Table 8.4 on page 330. The sum $(N_1s_1 + N_2s_2 + N_3s_3)$ is 85,615,456. Therefore, the optimal sampling fractions are: $f_1 = 37{,}108{,}776/85{,}615{,}456 = .4334$, $f_2 = 34{,}904{,}638/85{,}615{,}456 = .4077$, and $f_3 = 13{,}602{,}042/85{,}615{,}456 = .1589$. Next, consider the sample size formula

$$n = \frac{\sum_{i=1}^{k} N_i^2\sigma_i^2/f_i}{N^2D + \sum_{i=1}^{k} N_i\sigma_i^2}$$

It can be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for μ equal to E if we set D equal to $(E/z_{\alpha/2})^2$. It can also be shown that this sample size makes the margin of error in a $100(1 - \alpha)$ percent confidence interval for τ equal to E if we set D equal to $[E/(z_{\alpha/2}N)]^2$. For example, consider the department store chain situation and determining the sample size that makes the margin of error in a 95 percent confidence interval for the mean income per household equal to \$200. In this case, $D = (E/z_{\alpha/2})^2 = (200/1.96)^2 = 10{,}412.328$. Then, using the previously obtained optimal sampling fractions f_1, f_2, and f_3, and using s_1^2, s_2^2, and s_3^2 in Table 8.4 as estimates of σ_1^2, σ_2^2, and σ_3^2, we can make the following calculations:

$$\sum_{i=1}^{3} N_i^2 s_i^2 / f_i = 7.3299917 \times 10^{15} \qquad \left(N^2 D + \sum_{i=1}^{3} N_i s_i^2\right) = 5.5548425 \times 10^{12}$$

The needed sample size n is the ratio of these numbers. That is:

$$n = 7.3299917/5.5548425 \times 10^3 = 1320$$

The sizes of the samples to be selected from strata 1, 2, and 3 are then $n_1 = .4334n$, $n_2 = .4077n$, and $n_3 = .1589n$. Find these sample sizes.

Note: We have given the optimal allocation formulas and the sample size formulas for estimating μ and τ. When estimating p and τ, we use the same formulas that are used for μ and τ, except that we replace σ_i by $\sqrt{p_i(1 - p_i)}$ and σ_i^2 by $p_i(1 - p_i)$, where p_i is the true proportion for the ith stratum. Here, we can estimate $p_i(1 - p_i)$ by the results of a previously taken sample. For example, consider the department store chain situation and Table 8.4. Then, because $\hat{p}_1 = .5410$, $\hat{p}_2 = .7536$, and $\hat{p}_3 = .6667$, estimates of $p_1(1 - p_1)$, $p_2(1 - p_2)$, and $p_3(1 - p_3)$ are, respectively, .5410(.4590), .7536(.2464), and .6667(.3333).

Chapter Summary

In this chapter we discussed **confidence intervals** for population **means** and **proportions.** First, we studied how to compute a confidence interval for a **population mean.** We saw that when the population standard deviation σ is known, we can use the **normal distribution** to compute a confidence interval for a population mean. When σ is not known, if the population is normally distributed (or at least mound-shaped) or if the sample size n is large, we use the ***t* distribution** to compute this interval. We also studied how to find the size of the sample needed if we wish to compute a confidence interval for a mean with a prespecified *confidence level* and with a prespecified *margin of error.* Figure 8.18 is a flowchart summarizing our discussions concerning how to compute an appropriate confidence interval for a population mean.

Next we saw that we are often interested in estimating the proportion of population units falling into a category of interest. We showed how to compute a large sample confidence interval for a **population proportion,** and we saw how to find the sample size needed to estimate a population proportion. We concluded this chapter with an optional section that discusses how to estimate means, proportions, and totals for finite populations.

Glossary of Terms

confidence coefficient: The (before sampling) probability that a confidence interval for a population parameter will contain the population parameter. (page 304)

confidence interval: An interval of numbers computed so that we can be very confident (say, 95 percent confident) that a population parameter is contained in the interval. (page 301)

confidence level: The percentage of time that a confidence interval would contain a population parameter if all possible samples were used to calculate the interval. (pages 301, 303, and 306)

degrees of freedom (for a *t* curve): A parameter that describes the exact spread of the curve of a t distribution. (page 310)

margin of error: The quantity that is added to and subtracted from a point estimate of a population parameter to obtain a confidence interval for the parameter. (page 306)

population total: The sum of the values of all the population measurements. (page 328)

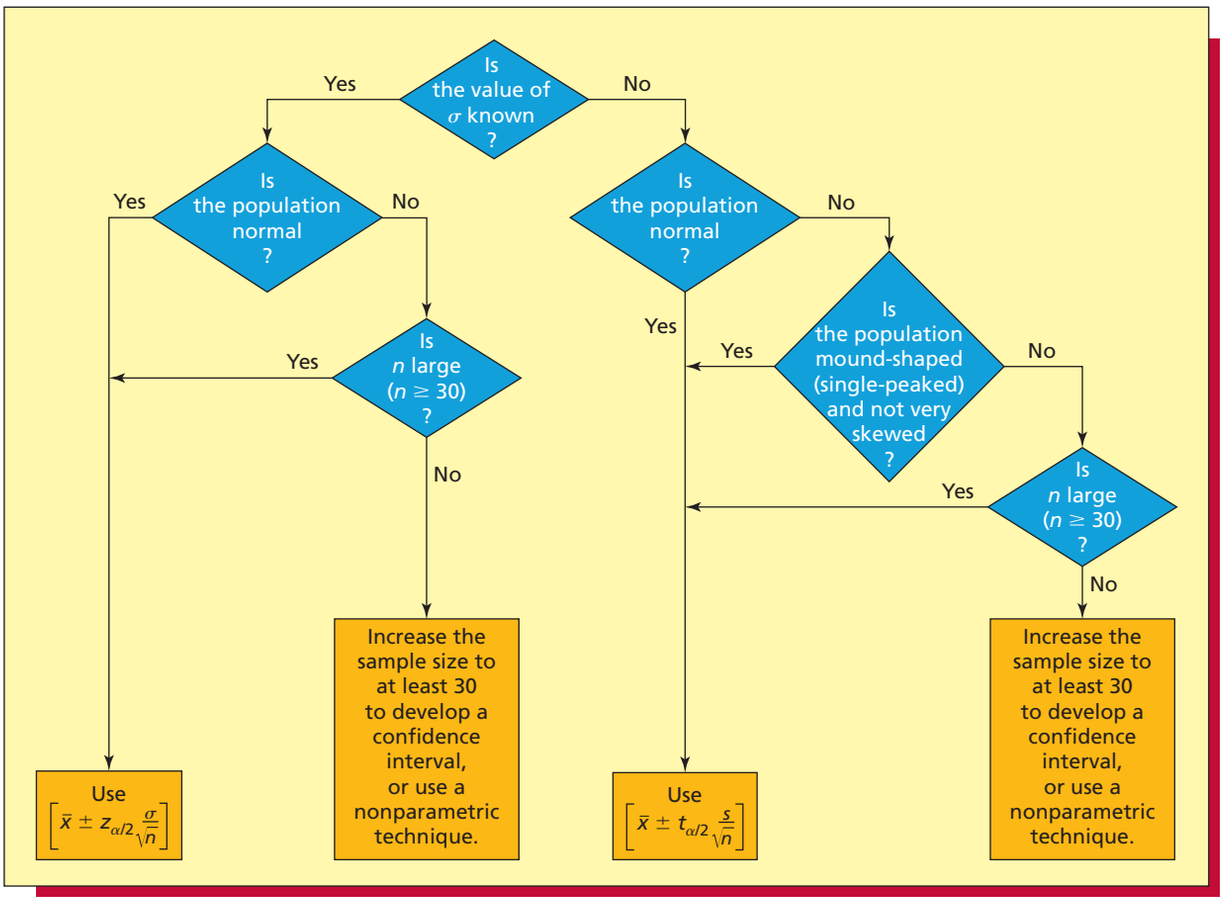
standard error of the estimate $\bar{x}$: The point estimate of $\sigma_{\bar{x}}$. (page 314)

***t* distribution:** A commonly used continuous probability distribution that is described by a distribution curve similar to a normal curve. The t curve is symmetrical about zero and is more spread out than a standard normal curve. (pages 310 and 311)

***t* point, t_α:** The point on the horizontal axis under a t curve that gives a right-hand tail area equal to α. (page 310)

***t* table:** A table of t point values listed according to the area in the tail of the t curve and according to values of the degrees of freedom. (pages 310–312)

FIGURE 8.18 Computing an Appropriate Confidence Interval for a Population Mean



Important Formulas

A z-based confidence interval for a population mean μ with σ known: page 307

A t-based confidence interval for a population mean μ with σ unknown: page 312

Sample size when estimating μ: page 318

A large sample confidence interval for a population proportion p: page 321

Sample size when estimating p: page 323

Confidence intervals for the population mean and total of a finite population: page 328

Confidence intervals for the proportion of and total number of units in a category (finite populations): page 329

Estimation in stratified random sampling: page 330

Supplementary Exercises

connect

8.57 In an article in the *Journal of Accounting Research,* Ashton, Willingham, and Elliott studied audit delay (the length of time from a company's fiscal year-end to the date of the auditor's report) for industrial and financial companies. In the study, a random sample of 250 industrial companies yielded a mean audit delay of 68.04 days with a standard deviation of 35.72 days, while a random sample of 238 financial companies yielded a mean audit delay of 56.74 days with a standard deviation of 34.87 days. Use these sample results to do the following:

a Calculate a 95 percent confidence interval for the mean audit delay for all industrial companies. Note: $t_{.025} = 1.97$ when $df = 249$.

b Calculate a 95 percent confidence interval for the mean audit delay for all financial companies. Note: $t_{.025} = 1.97$ when $df = 237$.

c By comparing the 95 percent confidence intervals you calculated in parts *a* and *b*, is there strong evidence that the mean audit delay for financial companies is shorter than the mean audit delay for industrial companies? Explain.

8.58 In an article in *Accounting and Business Research,* Beattie and Jones investigate the use and abuse of graphic presentations in the annual reports of United Kingdom firms. The authors found that 65 percent of the sampled companies graph at least one key financial variable, but that 30 percent of the graphics are materially distorted (nonzero vertical axis, exaggerated trend, or the like). Results for U.S. firms have been found to be similar.

a Suppose that in a random sample of 465 graphics from the annual reports of United Kingdom firms, 142 of the graphics are found to be distorted. Find a point estimate of and a 95 percent confidence interval for the proportion of all U.K. annual report graphics that are distorted.

b Based on this interval, can we be 95 percent confident that more than 25 percent of all graphics appearing in the annual reports of U.K. firms are distorted? Explain. Does this suggest that auditors should understand proper graphing methods?

c Determine the sample size needed in order to be 95 percent confident that $\hat{p}$, the sample proportion of U.K. annual report graphics that are distorted, is within a margin of error of .03 of p, the population proportion of all U.K. annual report graphics that are distorted.

8.59 THE DISK BRAKE CASE

National Motors has equipped the ZX-900 with a new disk brake system. We define the stopping distance for a ZX-900 to be the distance (in feet) required to bring the automobile to a complete stop from a speed of 35 mph under normal driving conditions using this new brake system. In addition, we define μ to be the mean stopping distance of all ZX-900s. One of the ZX-900's major competitors is advertised to achieve a mean stopping distance of 60 feet. National Motors would like to claim in a new advertising campaign that the ZX-900 achieves a shorter mean stopping distance.

Suppose that National Motors randomly selects a sample of $n = 81$ ZX-900s. The company records the stopping distance of each automobile and calculates the mean and standard deviation of the sample of $n = 81$ stopping distances to be $\bar{x} = 57.8$ ft and $s = 6.02$ ft.

a Calculate a 95 percent confidence interval for μ. Can National Motors be 95 percent confident that μ is less than 60 ft? Explain.

b Using the sample of $n = 81$ stopping distances as a preliminary sample, find the sample size necessary to make National Motors 95 percent confident that $\bar{x}$ is within a margin of error of one foot of μ.

8.60 In an article in the *Journal of Retailing,* J. G. Blodgett, D. H. Granbois, and R. G. Walters investigated negative word-of-mouth consumer behavior. In a random sample of 201 consumers, 150 reported that they engaged in negative word-of-mouth behavior (for instance, they vowed never to patronize a retailer again). In addition, the 150 respondents who engaged in such behavior, on average, told 4.88 people about their dissatisfying experience (with a standard deviation equal to 6.11).

a Use these sample results to compute a 95 percent confidence interval for the proportion of all consumers who engage in negative word-of-mouth behavior. On the basis of this interval, would it be reasonable to claim that more than 70 percent of all consumers engage in such behavior? Explain.

b Use the sample results to compute a 95 percent confidence interval for the population mean number of people who are told about a dissatisfying experience by consumers who engage in negative word-of-mouth behavior. On the basis of this interval, would it be reasonable to claim that these dissatisfied consumers tell, on average, at least three people about their bad experience? Explain. Note: $t_{.025} = 1.98$ when $df = 149$.

8.61 THE CIGARETTE ADVERTISEMENT CASE ModelAge

A random sample of 50 perceived age estimates for a model in a cigarette advertisement showed that $\bar{x} = 26.22$ years and that $s = 3.7432$ years.

a Use this sample to calculate a 95 percent confidence interval for the population mean age estimate for all viewers of the ad.

b Remembering that the cigarette industry requires that models must appear at least 25 years old, does the confidence interval make us 95 percent confident that the population mean perceived age estimate is at least 25? Is the mean perceived age estimate much more than 25? Explain.

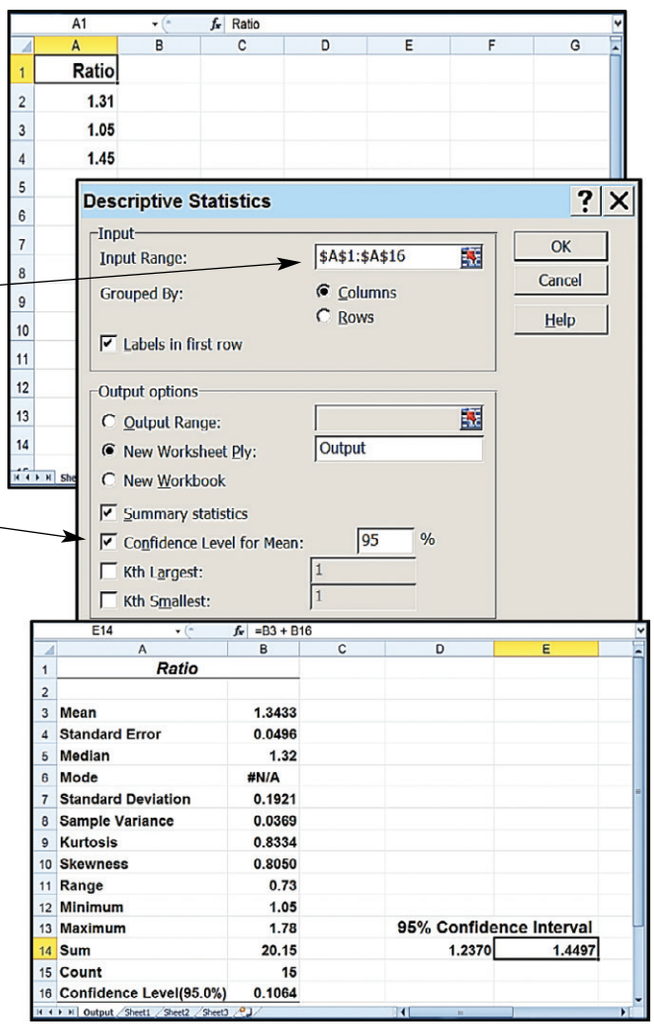
8.62 How safe are child car seats? *Consumer Reports* (May 2005) tested the safety of child car seats in 30 mph crashes. They found "slim safety margins" for some child car seats. Suppose that *Consumer Reports* simulates the safety of the market-leading child car seat. Their test consists of placing the maximum claimed weight in the car seat and simulating crashes at higher and higher miles per hour until a problem occurs. The following data identify the speed at which a problem with the car seat (such as the strap breaking, seat shell cracked, strap adjuster broke, detached from base, etc.) first appeared: 31.0, 29.4, 30.4, 28.9, 29.7, 30.1, 32.3, 31.7, 35.4, 29.1, 31.2, 30.2. Using the fact that $\bar{x} = 30.7833$ and $s = 1.7862$, find a 95 percent confidence interval for the mean speed at which a problem with the car seat first appears. Are we 95 percent confident that this mean is at least 30 mph? DS CarSeat

8.63 The manager of a chain of discount department stores wishes to estimate the total number of erroneous discounts allowed by sales clerks during the last month. A random sample of 1000 of the chain's 57,532 transactions for the last month reveals that erroneous discounts were allowed on 41 of the transactions. Use this sample information to find a point estimate of and a 95 percent confidence interval for the total number of erroneous discounts allowed during the last month.

Appendix 8.1 ■ Confidence Intervals Using Excel

Confidence interval for a population mean in Figure 8.12 on page 315 (data file: DebtEq.xlsx):

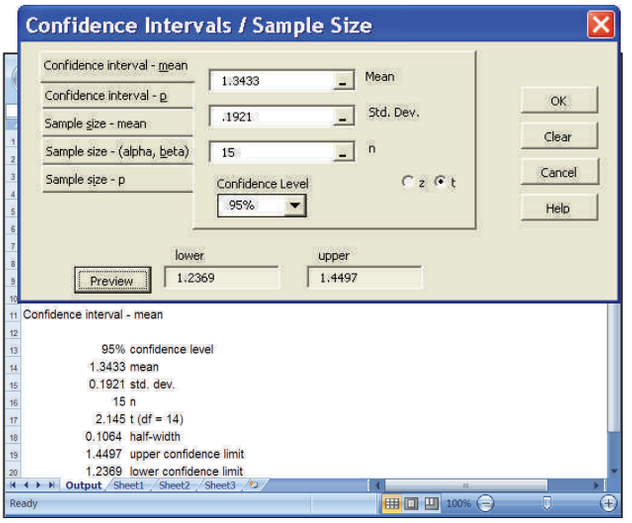
- Enter the debt-to-equity ratio data from Example 8.2 (page 313) into cells A2 to A16 with the label Ratio in cell A1.
- Select **Data : Data Analysis : Descriptive Statistics.**
- Click OK in the Data Analysis dialog box.
- In the Descriptive Statistics dialog box, enter A1:A16 into the Input Range window.
- Place a checkmark in the "Labels in first row" checkbox.
- Under output options, select "New Worksheet Ply" to have the output placed in a new worksheet and enter the name Output for the new worksheet.
- Place checkmarks in the Summary Statistics and "Confidence Level for Mean" checkboxes. This produces a *t*-based margin of error for a confidence interval.
- Type 95 in the "Confidence Level for Mean" box.
- Click OK in the Descriptive Statistics dialog box.
- A descriptive statistics summary will be displayed in cells A3 through B16 in the Output worksheet. Drag the column borders to reveal complete labels for all of the descriptive statistics.
- Type the heading "95% Confidence Interval" into cells D13 through E13.
- Compute the lower bound of the interval by typing the cell formula = B3 − B16 into cell D14. This subtracts the margin of error of the interval (labeled "Confidence Level (95%)") from the sample mean.
- Compute the upper bound of the interval by typing the formula = B3 + B16 into cell E14.



Appendix 8.2 ■ Confidence Intervals Using MegaStat

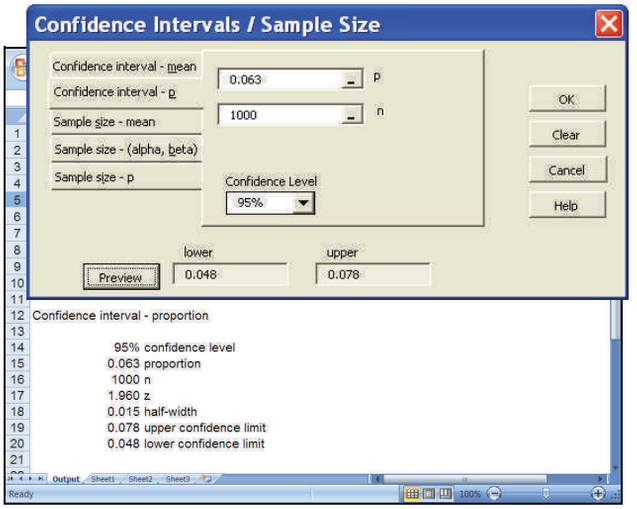
Confidence interval for the population mean debt-to-equity ratio in Example 8.2 on page 313:

- Select **Add-Ins : MegaStat : Confidence Intervals / Sample Size**
- In the "Confidence Intervals / Sample Size" dialog box, click on the "Confidence Interval—mean" tab.
- Enter the sample mean (here equal to 1.3433) into the Mean window.
- Enter the sample standard deviation (here equal to .1921) into the "Std Dev" window.
- Enter the sample size (here equal to 15) into the "n" window.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Select a *t*-based or *z*-based interval by clicking on "t" or "z." Here we request a *t*-based interval.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.



Confidence interval for a population proportion in the cheese spread situation of Example 8.6 on page 321:

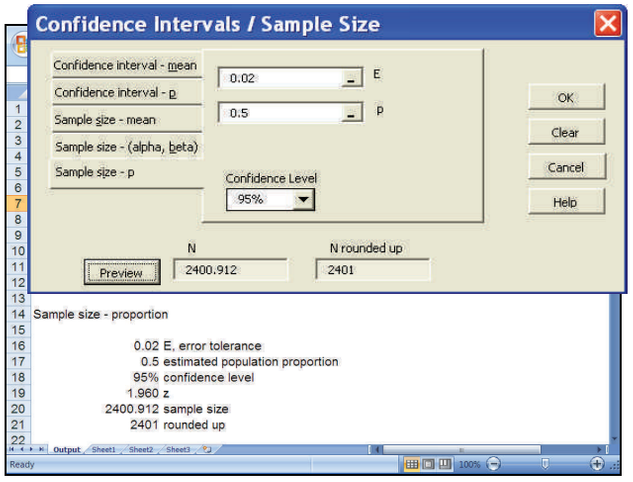
- In the "Confidence Intervals / Sample Size" dialog box, click on the "Confidence interval—p" tab.
- Enter the sample proportion (here equal to .063) into the "p" window.
- Enter the sample size (here equal to 1000) into the "n" window.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.



Sample size determination for a proportion problem on page 324:

- In the "Confidence Intervals / Sample Size" dialog box, click on the "Sample size—p" tab.
- Enter the desired margin of error (here equal to 0.02) into the "E" window and enter an estimate of the population proportion into the "p" window.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.

Sample size determination for a population mean problem is done by clicking on the "Sample Size—mean" tab. Then enter a desired margin of error, an estimate of the population standard deviation, and the desired level of confidence. Click OK.

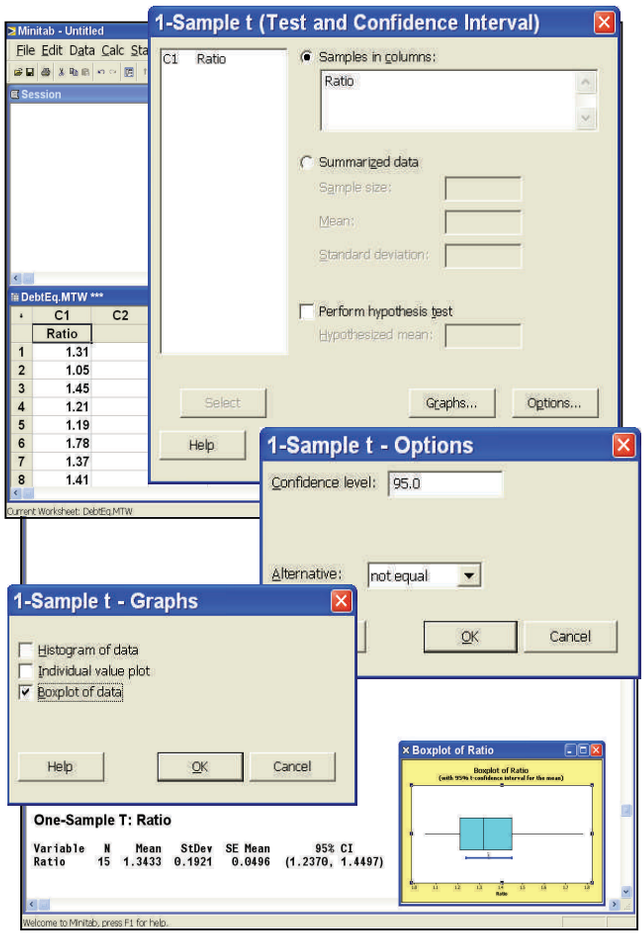


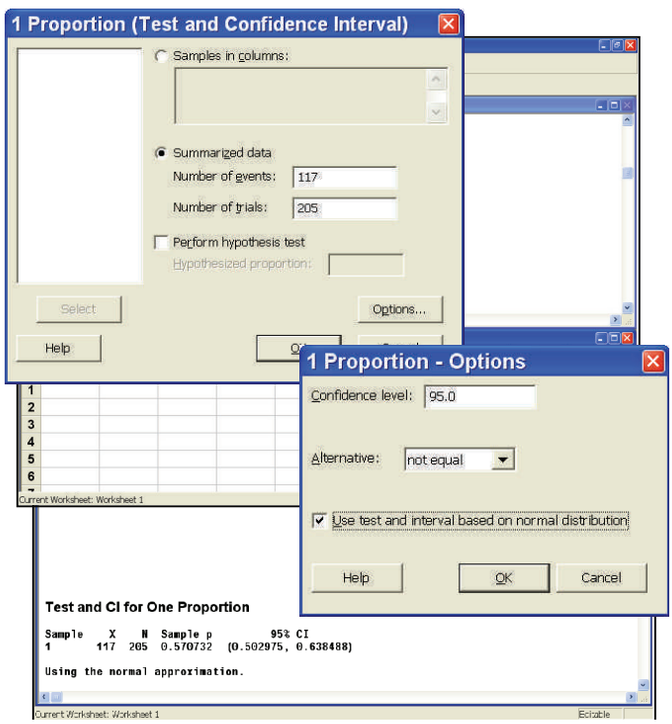
Appendix 8.3 ■ Confidence Intervals Using MINITAB

Confidence interval for a population mean in Figure 8.11 on page 315 (data file: DebtEq.MTW):

- In the Data window, enter the debt-to-equity ratio data from Example 8.2 (page 313) into a single column with variable name Ratio.
- Select **Stat : Basic Statistics : 1-Sample t.**
- In the "1-Sample t (Test and Confidence Interval)" dialog box, select "Samples in columns."
- Select the variable name Ratio into the "Samples in columns" window.
- Click the Options... button.
- In the "1-Sample t—Options" dialog box, enter the desired level of confidence (here 95.0) into the "Confidence level" window.
- Select "not equal" from the Alternative drop-down menu, and click OK in the "1-Sample t—Options" dialog box.
- To produce a boxplot of the data with a graphical representation of the confidence interval, click the Graphs... button, check the "Boxplot of data" checkbox, and click OK in the "1-Sample t—Graphs" dialog box.
- Click OK in "1-Sample t (Test and Confidence Interval)" dialog box.
- The confidence interval is given in the Session window, and the boxplot appears in a graphics window.

A "1-Sample Z" interval requiring a user-specified value of the population standard deviation is also available under Basic Statistics.



Confidence interval for a population proportion in the marketing ethics situation of Example 8.8 on pages 322 and 323:

- Select **Stat : Basic Statistics : 1 Proportion**
- In the "1 Proportion (Test and Confidence Interval)" dialog box, select "Summarized data."
- Enter the number of trials (here equal to 205) and the number of successes (or events) (here equal to 117) into the appropriate windows.
- Click on the Options... button.
- In the "1 Proportion—Options" dialog box, enter the desired level of confidence (here 95.0) into the "Confidence level" window.
- Select "not equal" from the Alternative drop-down menu.
- Check the "Use test and interval based on normal distribution" checkbox.
- Click OK in the "1 Proportion—Options" dialog box.
- Click OK in the "1 Proportion (Test and Confidence Interval)" dialog box.
- The confidence interval will be displayed in the Session window.