CHAPTER 6

Continuous Random Variables

Learning Objectives

After mastering the material in this chapter, you will be able to:

- **LO6-1** Define a continuous probability distribution and explain how it is used.
- **LO6-2** Use the uniform distribution to compute probabilities.
- **LO6-3** Describe the properties of the normal distribution and use a cumulative normal table.
- **LO6-4** Use the normal distribution to compute probabilities.
- **LO6-5** Find population values that correspond to specified normal distribution probabilities.
- **LO6-6** Use the normal distribution to approximate binomial probabilities (Optional).
- **LO6-7** Use the exponential distribution to compute probabilities (Optional).
- **LO6-8** Use a normal probability plot to help decide whether data come from a normal distribution (Optional).

Chapter Outline

In Chapter 5 we defined discrete and continuous random variables. We also discussed discrete probability distributions, which are used to compute the probabilities of values of discrete random variables. In this chapter we discuss **continuous probability distributions.** These are used to find probabilities concerning continuous random variables. We begin by explaining the general idea behind a continuous probability distribution. Then we present three important continuous distributions—**the uniform, normal,** and **exponential distributions.** We also study when and how the normal distribution can be used to approximate the binomial distribution (which was discussed in Chapter 5).

We will illustrate the concepts in this chapter by using two cases:

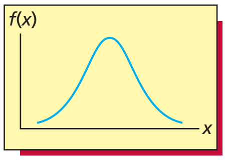
The Car Mileage Case: A competitor claims that its midsize car gets better mileage than an automaker's new midsize model. The automaker uses sample information and a probability based on the normal distribution to provide strong evidence that the competitor's claim is false.

The Coffee Temperature Case: A fast-food restaurant uses the normal distribution to estimate the proportion of coffee it serves that has a temperature (in degrees Fahrenheit) inside the range 153° to 167°, the customer requirement for best-tasting coffee.

6.1 Continuous Probability Distributions

LO6-1 Define a continuous probability distribution and explain how it is used.

We have said in Section 5.1 that when a random variable may assume any numerical value in one or more intervals on the real number line, then the random variable is called a **continuous random variable.** For example, as discussed in Section 5.1, the EPA combined city and highway mileage of a randomly selected midsize car is a continuous random variable. Furthermore, the temperature (in degrees Fahrenheit) of a randomly selected cup of coffee at a fast-food restaurant is also a continuous random variable. We often wish to compute probabilities about the range of values that a continuous random variable x might attain. For example, suppose that marketing research done by a fast-food restaurant indicates that coffee tastes best if its temperature is between 153° F and 167° F. The restaurant might then wish to find the probability that x, the temperature of a randomly selected cup of coffee at the restaurant, will be between 153° and 167°. This probability would represent the proportion of coffee served by the restaurant that has a temperature between 153° and 167°. Moreover, one minus this probability would represent the proportion of coffee served by the restaurant that has a temperature outside the range 153° to 167°.



In general, to compute probabilities concerning a continuous random variable x, we assign probabilities to **intervals of values** by using what we call a **continuous probability distribution.** To understand this idea, suppose that $f(x)$ is a continuous function of the numbers on the real line, and consider the continuous curve that results when $f(x)$ is graphed. Such a curve is illustrated in the figure on the page margin. Then:

Continuous Probability Distributions

The curve $f(x)$ is the **continuous probability distribution** of the random variable x if the probability that x will be in a specified interval of numbers is the area under the curve $f(x)$ corresponding to the interval. Sometimes we refer to a continuous probability distribution as a **probability curve** or as a **probability density function.**

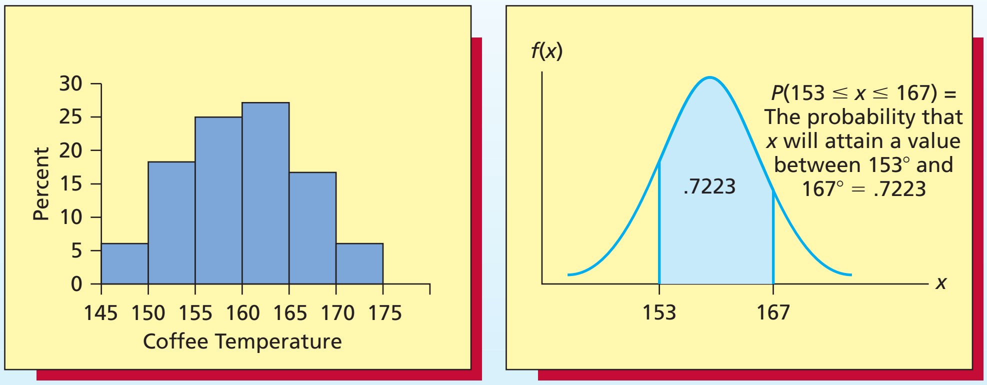
In this chapter we will study three continuous probability distributions—the *uniform, normal,* and *exponential* distributions. As an example of using a continuous probability distribution to describe a random variable, suppose that the fast-food restaurant will study the temperature of the coffee being dispensed at one of its locations. A temperature measurement is taken at a randomly selected time during each of the 24 half-hour periods from 8 A.M. to 7:30 P.M. on a given day. This is then repeated on a second day, giving the 48 coffee temperatures in Figure 6.1(a). Figure 6.1(b) shows a percent frequency histogram of the coffee temperatures. If we

FIGURE 6.1 The Coffee Temperature Data, Histogram, and Normal Curve

(a) The Coffee Temperature Data (Time Order Is Given By Reading Across And Then Down.)

154° F	165	148	157	160	157	152	149	171	168	165	164	156	151	161	157
154	159	155	153	173	164	161	151	158	160	153	161	160	158	169	163
146	167	162	159	166	158	173	162	155	150	165	154	160	162	159	166

(b) The Histogram **(c) The Normal Curve**



were to smooth out the histogram with a continuous curve, we would get a curve similar to the symmetrical and bell-shaped curve in Figure 6.1(c). One continuous probability distribution that graphs as a symmetrical and bell-shaped curve is the *normal probability distribution* (or *normal curve*). Because the coffee temperature histogram looks like a normal curve, it is reasonable to conclude that x, the temperature of a randomly selected cup of coffee at the fast-food restaurant, is described by a normal probability distribution. It follows that the probability that x will be between 153° and 167° is the area under the coffee temperature normal curve between 153 and 167. In Section 6.3, where we discuss the normal curve in detail, we will find that this area is .7223. That is, in probability notation: $P(153 \leq x \leq 167) = .7223$ (see the blue area in Figure 6.1(c)). In conclusion, we estimate that 72.23 percent of the coffee served at the restaurant is within the range of temperatures that is best and 27.77 percent of the coffee served is not in this range. If management wishes a very high percentage of the coffee served to taste best, it must improve the coffee-making process by better controlling temperatures.

We now present some general properties of a continuous probability distribution. We know that any probability is 0 or positive, and we also know that the probability assigned to all possible values of x must be 1. It follows that, similar to the conditions required for a discrete probability distribution, a probability curve must satisfy the following:

Properties of a Continuous Probability Distribution

The **continuous probability distribution** (or **probability curve**) $f(x)$ of a random variable x must satisfy the following two conditions:

1. $f(x) \geq 0$ for any value of x.
2. The total area under the curve $f(x)$ is equal to 1.

We have seen that to calculate a probability concerning a continuous random variable, we must compute an appropriate area under the curve $f(x)$. Because there is no area under a continuous curve at a single point, or number, on the real line, the probability that a continuous random variable x will equal a single numerical value is always equal to 0. It follows that if $[a, b]$ denotes an arbitrary interval of numbers on the real line, then $P(x = a) = 0$ and $P(x = b) = 0$. Therefore, $P(a \leq x \leq b)$ equals $P(a < x < b)$ because each of the interval endpoints a and b has a probability that is equal to 0.

6.2 The Uniform Distribution

LO6-2 Use the uniform distribution to compute probabilities.

Suppose that over a period of several days the manager of a large hotel has recorded the waiting times of 1,000 people waiting for an elevator in the lobby at dinnertime (5:00 P.M. to 7:00 P.M.). The observed waiting times range from zero to four minutes. Furthermore, when the waiting times are arranged into a histogram, the bars making up the histogram have approximately equal heights, giving the histogram a rectangular appearance. This implies that the relative frequencies of all waiting times from zero to four minutes are about the same. Therefore, it is reasonable to use the continuous *uniform distribution* to describe the random variable x, the amount of time a randomly selected hotel patron spends waiting for an elevator. The equation describing the uniform distribution in this situation is

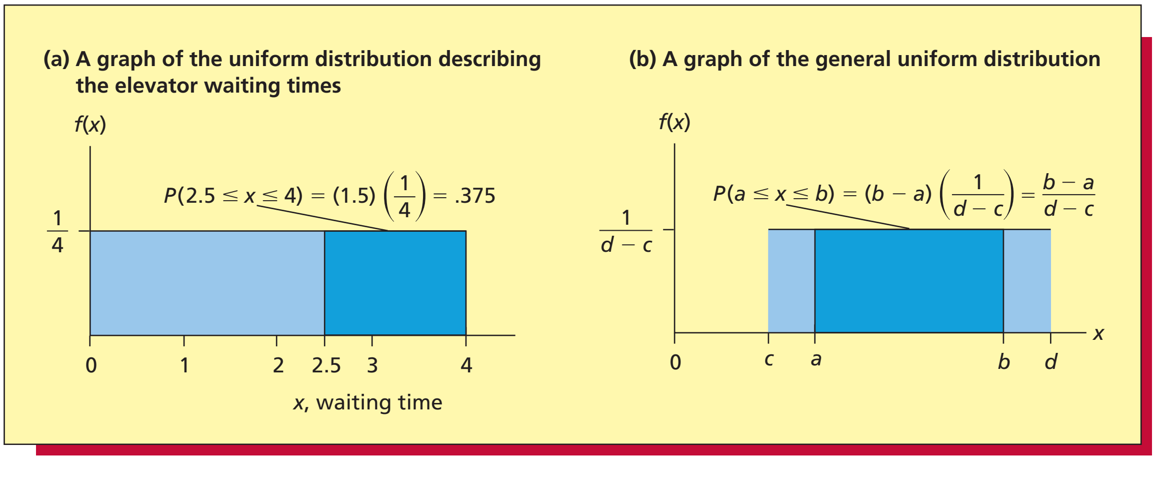
$$f(x) = \begin{cases} \dfrac{1}{4} & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Noting that this equation is graphed in Figure 6.2(a), suppose that the hotel manager feels that an elevator waiting time of 2.5 minutes or more is unacceptably long. Therefore, to find the probability that a hotel patron will wait too long, the manager wishes to find the probability that a randomly selected patron will spend at least 2.5 minutes waiting for an elevator. This probability is the area under the curve $f(x)$ that corresponds to the interval [2.5, 4]. As shown in Figure 6.2(a), this probability is the area of a rectangle having a base equal to $4 - 2.5 = 1.5$ and a height equal to $1/4$. That is,

$$P(x \geq 2.5) = P(2.5 \leq x \leq 4) = \text{base} \times \text{height} = 1.5 \times \frac{1}{4} = .375$$

This says that 37.5 percent of all hotel patrons will spend at least 2.5 minutes waiting for an elevator at dinnertime. Based on this result, the hotel manager would probably decide that too many patrons are waiting too long for an elevator and that action should be taken to reduce elevator waiting times.

FIGURE 6.2 The Uniform Distribution



In general, the equation that describes the uniform distribution is given in the following box and is graphed in Figure 6.2(b).

The Uniform Distribution

If c and d are numbers on the real line, the equation describing the **uniform distribution** is

$$f(x) = \begin{cases} \dfrac{1}{d-c} & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, the mean and the standard deviation of the population of all possible observed values of a random variable x that has a uniform distribution are

$$\mu_x = \frac{c+d}{2} \qquad \text{and} \qquad \sigma_x = \frac{d-c}{\sqrt{12}}$$

Notice that the total area under the uniform distribution is the area of a rectangle having a base equal to $(d - c)$ and a height equal to $1/(d - c)$. Therefore, the probability curve's total area is

$$\text{base} \times \text{height} = (d-c)\left(\frac{1}{d-c}\right) = 1$$

(remember that the total area under any continuous probability curve must equal 1). Furthermore, if a and b are numbers that are as illustrated in Figure 6.2(b), then the probability that x will be between a and b is the area of a rectangle with base $(b - a)$ and height $1/(d - c)$. That is,

$$P(a \le x \le b) = \text{base} \times \text{height} = (b-a)\left(\frac{1}{d-c}\right) = \frac{b-a}{d-c}$$

EXAMPLE 6.1 Elevator Waiting Times

In the introduction to this section we have said that the amount of time, x, that a randomly selected hotel patron spends waiting for the elevator at dinnertime is uniformly distributed between zero and four minutes. In this case, $c = 0$ and $d = 4$. Therefore,

$$f(x) = \begin{cases} \dfrac{1}{d-c} = \dfrac{1}{4-0} = \dfrac{1}{4} & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Moreover, the mean waiting time for the elevator is

$$\mu_x = \frac{c+d}{2} = \frac{0+4}{2} = 2 \text{ (minutes)}$$

and the standard deviation of the waiting times is

$$\sigma_x = \frac{d-c}{\sqrt{12}} = \frac{4-0}{\sqrt{12}} = 1.1547 \text{ (minutes)}$$

Therefore, noting that $\mu_x - \sigma_x = 2 - 1.1547 = .8453$ and that $\mu_x + \sigma_x = 2 + 1.1547 = 3.1547$, the probability that the waiting time of a randomly selected patron will be within (plus or minus) one standard deviation of the mean waiting time is

$$P(.8453 \leq x \leq 3.1547) = (3.1547 - .8453) \times \frac{1}{4}$$
$$= .57735$$

Exercises for Sections 6.1 and 6.2

CONCEPTS

6.1 A discrete probability distribution assigns probabilities to individual values. To what are probabilities assigned by a continuous probability distribution?

6.2 How do we use the continuous probability distribution (or probability curve) of a random variable x to find probabilities? Explain.

6.3 What two properties must be satisfied by a continuous probability distribution (or probability curve)?

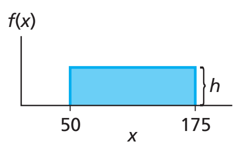
6.4 Is the height of a probability curve over a given point a probability? Explain.

6.5 When is it appropriate to use the uniform distribution to describe a random variable x?

METHODS AND APPLICATIONS

6.6 Suppose that the random variable x has a uniform distribution with $c = 2$ and $d = 8$.

- **a** Write the formula for the probability curve of x, and write an interval that gives the possible values of x.
- **b** Graph the probability curve of x.
- **c** Find $P(3 \leq x \leq 5)$.
- **d** Find $P(1.5 \leq x \leq 6.5)$.
- **e** Calculate the mean μ_x, variance σ_x^2, and standard deviation σ_x.
- **f** Calculate the interval $[\mu_x \pm 2\sigma_x]$. What is the probability that x will be in this interval?



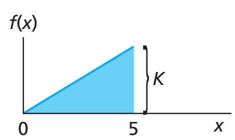
6.7 Consider the figure given in the margin. Find the value h that makes the function $f(x)$ a valid continuous probability distribution.

6.8 Assume that the waiting time x for an elevator is uniformly distributed between zero and six minutes.

- **a** Write the formula for the probability curve of x.
- **b** Graph the probability curve of x.
- **c** Find $P(2 \leq x \leq 4)$.
- **d** Find $P(3 \leq x \leq 6)$.
- **e** Find $P(\{0 \leq x \leq 2\}$ or $\{5 \leq x \leq 6\})$.

6.9 Refer to Exercise 6.8.

- **a** Calculate the mean, μ_x, the variance, σ_x^2, and the standard deviation, σ_x.
- **b** Find the probability that the waiting time of a randomly selected patron will be within one standard deviation of the mean.



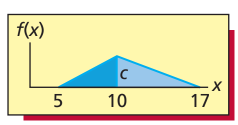
6.10 Consider the figure given in the margin. Find the value k that makes the function $f(x)$ a valid continuous probability distribution.

6.11 Suppose that an airline quotes a flight time of 2 hours, 10 minutes between two cities. Furthermore, suppose that historical flight records indicate that the actual flight time between the two cities, x, is uniformly distributed between 2 hours and 2 hours, 20 minutes. Letting the time unit be one minute,

- **a** Write the formula for the probability curve of x.
- **b** Graph the probability curve of x.
- **c** Find $P(125 \leq x \leq 135)$.
- **d** Find the probability that a randomly selected flight between the two cities will be at least five minutes late.

6.12 Refer to Exercise 6.11.

- **a** Calculate the mean flight time and the standard deviation of the flight time.
- **b** Find the probability that the flight time will be within one standard deviation of the mean.



6.13 Consider the figure given in the margin. Find the value c that makes the function $f(x)$ a valid continuous probability distribution.

6.14 A weather forecaster predicts that the May rainfall in a local area will be between three and six inches but has no idea where within the interval the amount will be. Let x be the amount of May rainfall in the local area, and assume that x is uniformly distributed over the interval three to six inches.

a Write the formula for the probability curve of x and graph this probability curve.

b What is the probability that May rainfall will be at least four inches? At least five inches?

6.15 Refer to Exercise 6.14 and find the probability that the observed May rainfall will fall within two standard deviations of the mean May rainfall.

LO6-3 Describe the properties of the normal distribution and use a cumulative normal table.

6.3 The Normal Probability Distribution

The normal curve The bell-shaped appearance of the normal probability distribution is illustrated in Figure 6.3. The equation that defines this normal curve is given in the following box:

The Normal Probability Distribution

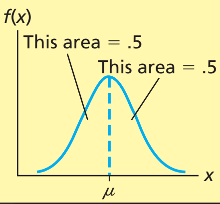
The **normal probability distribution** is defined by the equation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for all values of } x \text{ on the real line}$$

Here μ and σ are the mean and standard deviation of the population of all possible observed values of the random variable x under consideration. Furthermore, $\pi = 3.14159\ldots$, and $e = 2.71828\ldots$ is the base of Napierian logarithms.

FIGURE 6.3 The Normal Probability Curve

The normal curve is symmetrical around μ, and the total area under the curve equals 1.



Although this equation looks very intimidating, we will not use it to find areas (and thus probabilities) under the normal curve. Instead, we will use a *normal curve table*. What is important to know for now is that the normal probability distribution has several important properties:

1. There is an entire family of normal probability distributions; the specific shape of each normal distribution is determined by its mean μ and its standard deviation σ.
2. The highest point on the normal curve is located at the mean, which is also the median and the mode of the distribution.
3. The normal distribution is symmetrical: The curve's shape to the left of the mean is the mirror image of its shape to the right of the mean.
4. The tails of the normal curve extend to infinity in both directions and never touch the horizontal axis. However, the tails get close enough to the horizontal axis quickly enough to ensure that the total area under the normal curve equals 1.
5. Because the normal curve is symmetrical, the area under the normal curve to the right of the mean (μ) equals the area under the normal curve to the left of the mean, and each of these areas equals .5 (see Figure 6.3).

Intuitively, the mean μ positions the normal curve on the real line. This is illustrated in Figure 6.4(a). This figure shows two normal curves with different means μ_1 and μ_2 (where μ_1 is greater than μ_2) and with equal standard deviations. We see that the normal curve with mean μ_1 is centered farther to the right.

The variance σ^2 (and the standard deviation σ) measure the spread of the normal curve. This is illustrated in Figure 6.4(b), which shows two normal curves with the same mean and two different standard deviations σ_1 and σ_2. Because σ_1 is greater than σ_2, the normal curve with standard deviation σ_1 is more spread out (flatter) than the normal curve with standard deviation σ_2. In general, larger standard deviations result in normal curves that are flatter and more spread out, while smaller standard deviations result in normal curves that have higher peaks and are less spread out.

Suppose that a random variable x is described by a normal probability distribution (or is, as we say, **normally distributed**) with mean μ and standard deviation σ. If a and b are numbers on the

FIGURE 6.4 How the Mean μ and Standard Deviation σ Affect the Position and Shape of a Normal Probability Curve

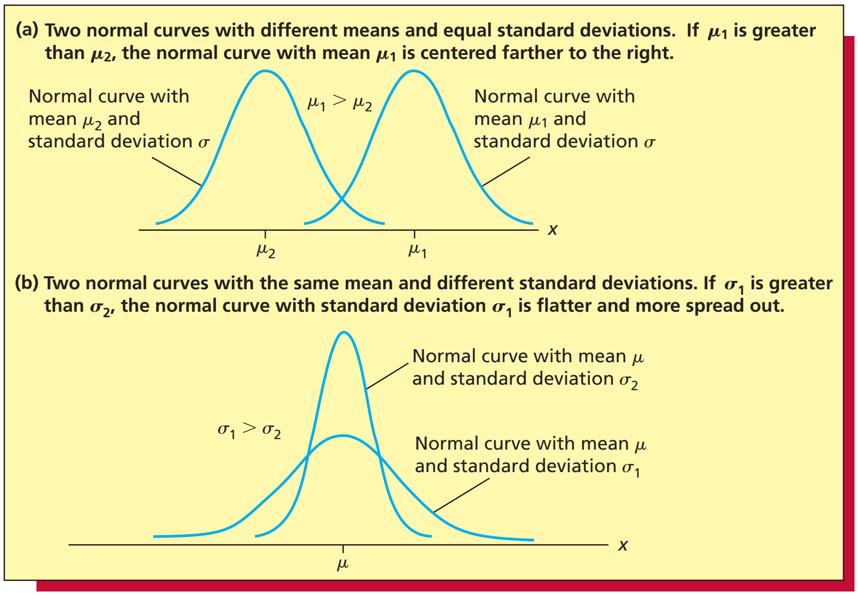


FIGURE 6.5 An Area under a Normal Curve Corresponding to the Interval [*a*, *b*]

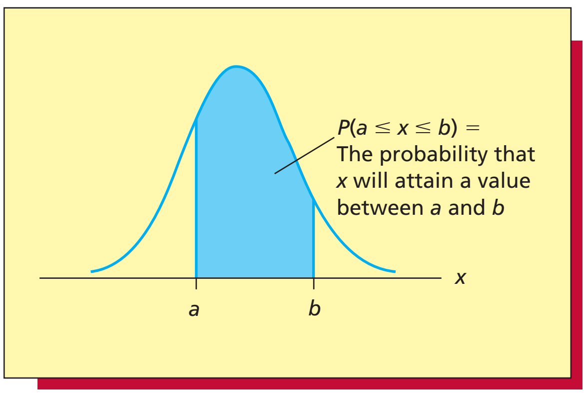
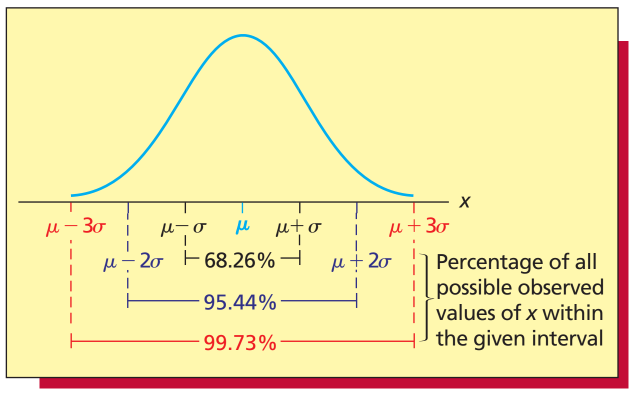


FIGURE 6.6 Three Important Percentages Concerning a Normally Distributed Random Variable x with Mean μ and Standard Deviation σ



real line, we consider the probability that x will be between a and b. That is, we consider

$$P(a \leq x \leq b)$$

which equals the area under the normal curve with mean μ and standard deviation σ corresponding to the interval $[a, b]$. Such an area is depicted in Figure 6.5. We soon explain how to find such areas using a statistical table called a **normal table.** For now, we emphasize three important areas under a normal curve. These areas form the basis for the **Empirical Rule** for a normally distributed population. Specifically, if x is normally distributed with mean μ and standard deviation σ, it can be shown (using a normal table) that, as illustrated in Figure 6.6:

Three Important Areas under the Normal Curve

1. $P(\mu - \sigma \leq x \leq \mu + \sigma) = .6826$

   This means that 68.26 percent of all possible observed values of x are within (plus or minus) one standard deviation of μ.

2. $P(\mu - 2\sigma \leq x \leq \mu + 2\sigma) = .9544$

   This means that 95.44 percent of all possible observed values of x are within (plus or minus) two standard deviations of μ.

3. $P(\mu - 3\sigma \leq x \leq \mu + 3\sigma) = .9973$

   This means that 99.73 percent of all possible observed values of x are within (plus or minus) three standard deviations of μ.

Finding normal curve areas There is a unique normal curve for every combination of μ and σ. Because there are many (theoretically, an unlimited number of) such combinations, we would like to have one table of normal curve areas that applies to all normal curves. There is such a table, and we can use it by thinking in terms of how many standard deviations a value of interest is from the mean. Specifically, consider a random variable x that is normally distributed with mean μ and standard deviation σ. Then the random variable

$$z = \frac{x - \mu}{\sigma}$$

expresses the number of standard deviations that x is from the mean μ. To understand this idea, notice that if x equals μ (that is, x is zero standard deviations from μ), then $z = (\mu - \mu)/\sigma = 0$. However, if x is one standard deviation above the mean (that is, if x equals $\mu + \sigma$), then $x - \mu = \sigma$ and $z = \sigma/\sigma = 1$. Similarly, if x is two standard deviations below the mean (that is, if x equals $\mu - 2\sigma$), then $x - \mu = -2\sigma$ and $z = -2\sigma/\sigma = -2$. Figure 6.7 illustrates that for values of x of, respectively, $\mu - 3\sigma$, $\mu - 2\sigma$, $\mu - \sigma$, μ, $\mu + \sigma$, $\mu + 2\sigma$, and $\mu + 3\sigma$, the corresponding values of z are $-3, -2, -1, 0, 1, 2,$ and 3. This figure also illustrates the following general result:

The Standard Normal Distribution

If a random variable x (or, equivalently, the population of all possible observed values of x) is normally distributed with mean μ and standard deviation σ, then the random variable

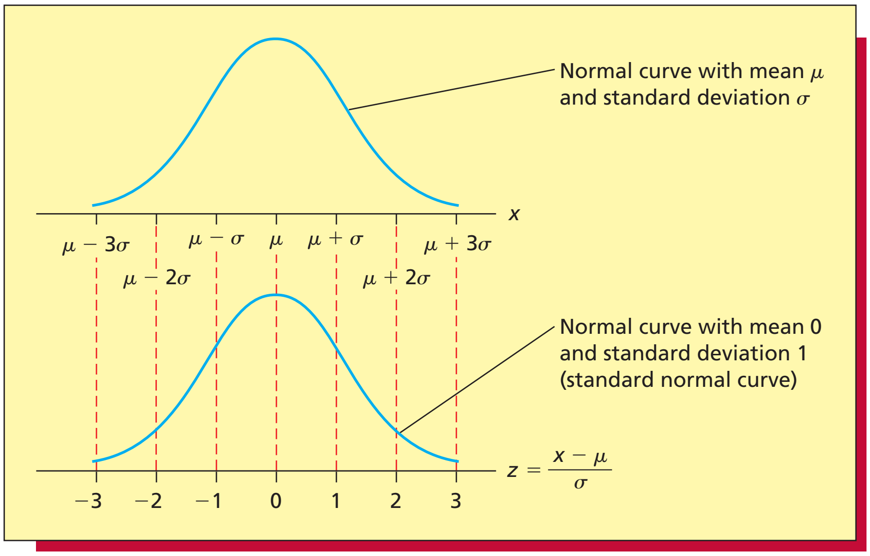
$$z = \frac{x - \mu}{\sigma}$$

(or, equivalently, the population of all possible observed values of z) is normally distributed with mean 0 and standard deviation 1. A normal distribution (or curve) with mean 0 and standard deviation 1 is called a **standard normal distribution** (or **curve**).

Table A.3 (on pages 790 and 791) is a table of *cumulative* areas under the standard normal curve. This table is called a *cumulative normal table,* and it is reproduced as Table 6.1 (on pages 233 and 234). Specifically,

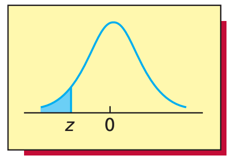
The **cumulative normal table** gives, for many different values of z, the area under the standard normal curve to the left of z.

FIGURE 6.7 **If x Is Normally Distributed with Mean μ and Standard Deviation σ, Then $z = \frac{x-\mu}{\sigma}$ Is Normally Distributed with Mean 0 and Standard Deviation 1**

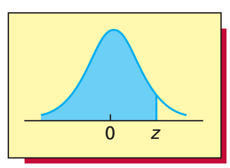


Two such areas are shown next to Table 6.1—one with a negative z value and one with a positive z value. The values of z in the cumulative normal table range from -3.99 to 3.99 in increments of .01. As can be seen from Table 6.1, values of z accurate to the nearest tenth are given in the far left column (headed z) of the table. Further graduations to the nearest hundredth (.00, .01, .02, . . . , .09) are given across the top of the table. The areas under the normal curve are given in the body of the table, accurate to four (or sometimes five) decimal places.

TABLE 6.1 Cumulative Areas under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
−3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
−3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
−3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
−3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
−3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
−3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
−3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
−3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
−3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2482	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389



(Table Continues)

TABLE 6.1 Cumulative Areas under the Standard Normal Curve (*Continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99897	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

As an example, suppose that we wish to find the area under the standard normal curve to the left of a z value of 2.00. This area is illustrated in Figure 6.8. To find this area, we start at the top of the leftmost column in Table 6.1 (previous page) and scan down the column past the negative z values. We then scan through the positive z values (which continue on the top of this page) until we find the z value 2.0—see the red arrow above. We now scan across the row in the table corresponding to the z value 2.0 until we find the column corresponding to the heading .00. The desired area (which we have shaded blue) is in the row corresponding to the z value 2.0 and in the column headed .00. This area, which equals .9772, is the probability that the random variable z will be less than or equal to 2.00. That is, we have found that $P(z \leq 2) = .9772$. Note that, because there is no area under the normal curve at a single value of z, there is no difference between $P(z \leq 2)$ and $P(z < 2)$. As another example, the area under the standard normal curve to the left of the z value 1.25 is found in the row corresponding to 1.2 and in the column corresponding to .05. We find that this area (also shaded blue) is .8944. That is, $P(z \leq 1.25) = .8944$ (see Figure 6.9).

We now show how to use the cumulative normal table to find several other kinds of normal curve areas. First, suppose that we wish to find the area under the standard normal curve to the right of a z value of 2—that is, we wish to find $P(z \geq 2)$. This area is illustrated in Figure 6.10 and is called a **right-hand tail area.** Because the total area under the normal curve equals 1, the area under the curve to the right of 2 equals 1 minus the area under the curve to the left of 2. Because Table 6.1 tells us that the area under the standard normal curve to the left of 2 is .9772, the area under the standard normal curve to the right of 2 is $1 - .9772 = .0228$. Said in an equivalent fashion, because $P(z \leq 2) = .9772$, it follows that $P(z \geq 2) = 1 - P(z \leq 2) = 1 - .9772 = .0228$.

FIGURE 6.8 Finding $P(z \leq 2)$

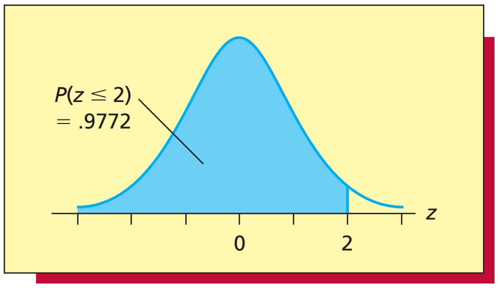


FIGURE 6.9 Finding $P(z \leq 1.25)$

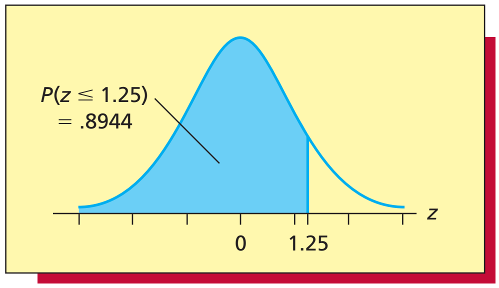


FIGURE 6.10 Finding $P(z \geq 2)$

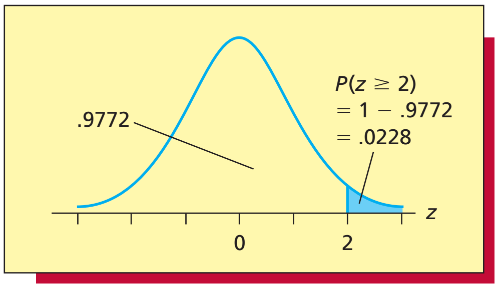
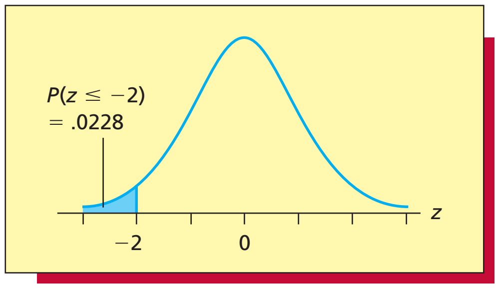


FIGURE 6.11 Finding $P(z \leq -2)$



Next, suppose that we wish to find the area under the standard normal curve to the left of a z value of -2. That is, we wish to find $P(z \leq -2)$. This area is illustrated in Figure 6.11 and is called a **left-hand tail area.** The needed area is found in the row of the cumulative normal table corresponding to -2.0 (on page 233) and in the column headed by .00. We find that $P(z \leq -2) = .0228$. Notice that the area under the standard normal curve to the left of -2 is equal to the area under this curve to the right of 2. This is true because of the symmetry of the normal curve.

Figure 6.12 illustrates how to find the area under the standard normal curve to the right of -2. Because the total area under the normal curve equals 1, the area under the curve to the right of -2 equals 1 minus the area under the curve to the left of -2. Because Table 6.1 tells us that the area under the standard normal curve to the left of -2 is .0228, the area under the standard normal curve to the right of -2 is $1 - .0228 = .9772$. That is, because $P(z \leq -2) = .0228$, it follows that $P(z \geq -2) = 1 - P(z \leq -2) = 1 - .0228 = .9772$.

The smallest z value in Table 6.1 is -3.99, and the table tells us that the area under the standard normal curve to the left of -3.99 is .00003 (see Figure 6.13). Therefore, if we wish to find the area under the standard normal curve to the left of any z value less than -3.99, the most we can say (without using a computer) is that this area is less than .00003. Similarly, the area under

FIGURE 6.12 Finding $P(z \geq -2)$

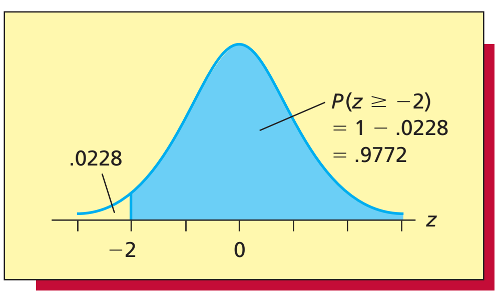


FIGURE 6.13 Finding $P(z \leq -3.99)$

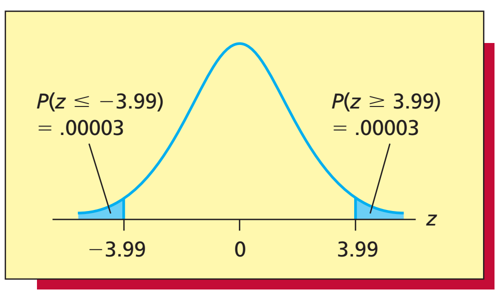
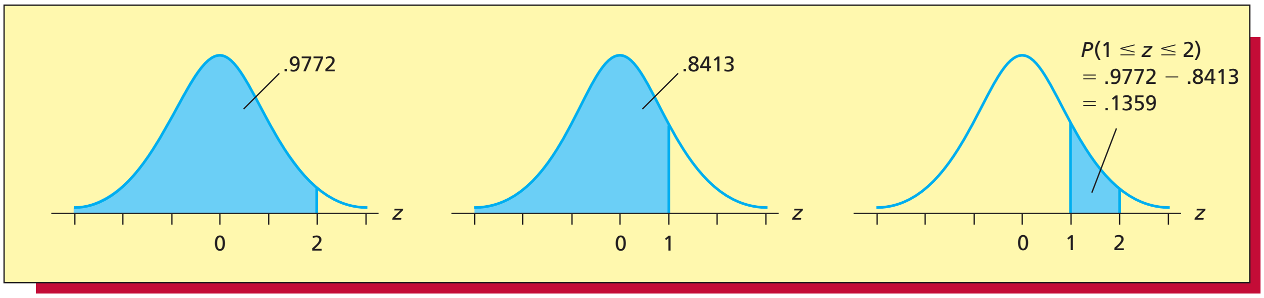


FIGURE 6.14 **Calculating $P(1 \le z \le 2)$**

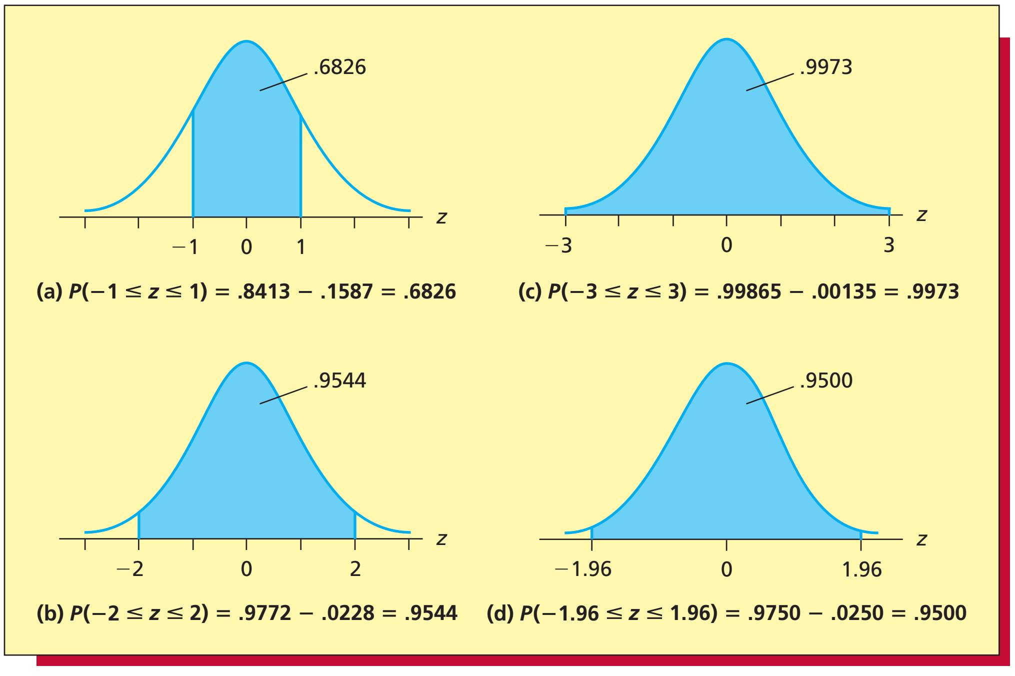


the standard normal curve to the right of any z value greater than 3.99 is also less than .00003 (see Figure 6.13).

Figure 6.14 illustrates how to find the area under the standard normal curve between 1 and 2. This area equals the area under the curve to the left of 2, which the normal table tells us is .9772, minus the area under the curve to the left of 1, which the normal table tells us is .8413. Therefore, $P(1 \le z \le 2) = .9772 - .8413 = .1359$.

To conclude our introduction to using the normal table, we will use this table to justify the Empirical Rule. Figure 6.15(a) illustrates the area under the standard normal curve between -1 and 1. This area equals the area under the curve to the left of 1, which the normal table tells us is .8413, minus the area under the curve to the left of -1, which the normal table tells us is .1587. Therefore, $P(-1 \le z \le 1) = .8413 - .1587 = .6826$. Now, suppose that a random variable x is normally distributed with mean μ and standard deviation σ, and remember that z is the number of standard deviations σ that x is from μ. It follows that when we say that $P(-1 \le z \le 1)$ equals .6826, we are saying that 68.26 percent of all possible observed values of x are between a point that is one standard deviation below μ (where z equals -1) and a point that is one standard deviation

FIGURE 6.15 **Some Areas under the Standard Normal Curve**



(a) $P(-1 \le z \le 1) = .8413 - .1587 = .6826$

(c) $P(-3 \le z \le 3) = .99865 - .00135 = .9973$

(b) $P(-2 \le z \le 2) = .9772 - .0228 = .9544$

(d) $P(-1.96 \le z \le 1.96) = .9750 - .0250 = .9500$

above μ (where z equals 1). That is, 68.26 percent of all possible observed values of x are within (plus or minus) one standard deviation of the mean μ.

Figure 6.15(b) illustrates the area under the standard normal curve between -2 and 2. This area equals the area under the curve to the left of 2, which the normal table tells us is .9772, minus the area under the curve to the left of -2, which the normal table tells us is .0228. Therefore, $P(-2 \leq z \leq 2) = .9772 - .0228 = .9544$. That is, 95.44 percent of all possible observed values of x are within (plus or minus) two standard deviations of the mean μ.

Figure 6.15(c) illustrates the area under the standard normal curve between -3 and 3. This area equals the area under the curve to the left of 3, which the normal table tells us is .99865, minus the area under the curve to the left of -3, which the normal table tells us is .00135. Therefore, $P(-3 \leq z \leq 3) = .99865 - .00135 = .9973$. That is, 99.73 percent of all possible observed values of x are within (plus or minus) three standard deviations of the mean μ.

Although the Empirical Rule gives the percentages of all possible values of a normally distributed random variable x that are within one, two, and three standard deviations of the mean μ, we can use the normal table to find the percentage of all possible values of x that are within any particular number of standard deviations of μ. For example, in later chapters we will need to know the percentage of all possible values of x that are within plus or minus 1.96 standard deviations of μ. Figure 6.15(d) illustrates the area under the standard normal curve between -1.96 and 1.96. This area equals the area under the curve to the left of 1.96, which the normal table tells us is .9750, minus the area under the curve to the left of -1.96, which the table tells us is .0250. Therefore, $P(-1.96 \leq z \leq 1.96) = .9750 - .0250 = .9500$. That is, 95 percent of all possible values of x are within plus or minus 1.96 standard deviations of the mean μ.

Some practical applications We have seen how to use z values and the normal table to find areas under the standard normal curve. However, most practical problems are not stated in such terms. We now consider an example in which we must restate the problem in terms of the standard normal random variable z before using the normal table.

LO6-4 Use the normal distribution to compute probabilities.

EXAMPLE 6.2 The Car Mileage Case: Estimating Mileage

C

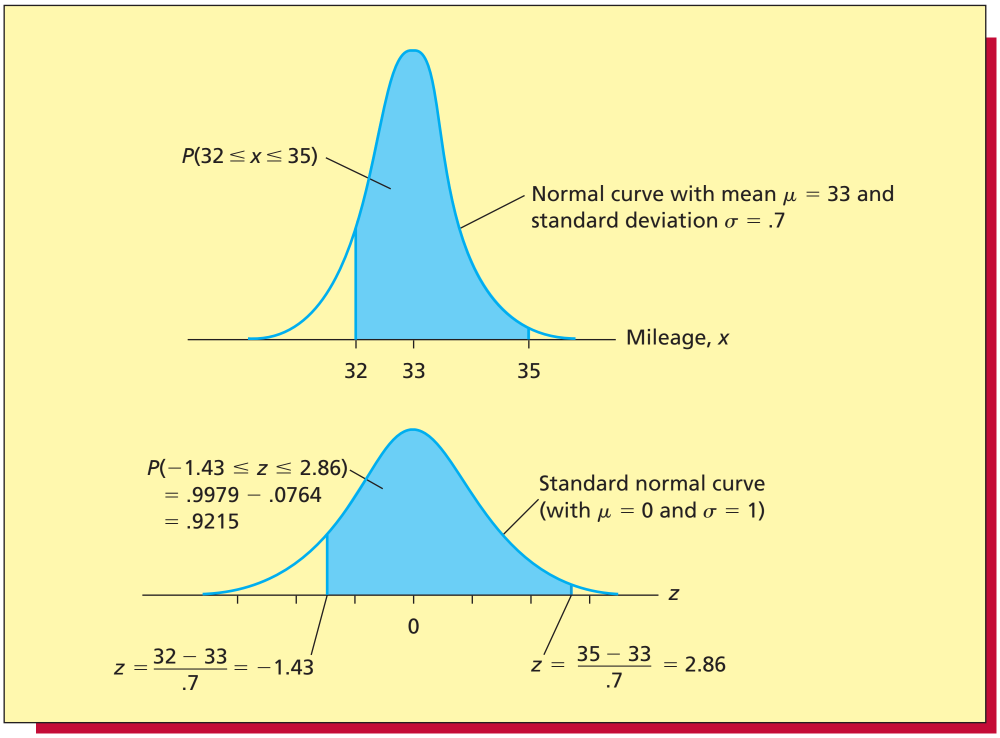
Recall from previous chapters that an automaker has recently introduced a new midsize model and that we have used the sample of 50 mileages to estimate that the population of mileages of all cars of this type is normally distributed with a mean mileage equal to 31.56 mpg and a standard deviation equal to .798 mpg. Suppose that a competing automaker produces a midsize model that is somewhat smaller and less powerful than the new midsize model. The competitor claims, however, that its midsize model gets better mileages. Specifically, the competitor claims that the mileages of all its midsize cars are normally distributed with a mean mileage μ equal to 33 mpg and a standard deviation σ equal to .7 mpg. In the next example we consider one way to investigate the validity of this claim. In this example we assume that the claim is true, and we calculate the probability that the mileage, x, of a randomly selected competing midsize car will be between 32 mpg and 35 mpg. That is, we wish to find $P(32 \leq x \leq 35)$. As illustrated in Figure 6.16 on the next page, this probability is the area between 32 and 35 under the normal curve having mean $\mu = 33$ and standard deviation $\sigma = .7$. In order to use the normal table, we must restate the problem in terms of the standard normal random variable z. The z value corresponding to 32 is

$$z = \frac{x - \mu}{\sigma} = \frac{32 - 33}{.7} = \frac{-1}{.7} = -1.43$$

which says that the mileage 32 is 1.43 standard deviations below the mean $\mu = 33$. The z value corresponding to 35 is

$$z = \frac{x - \mu}{\sigma} = \frac{35 - 33}{.7} = \frac{2}{.7} = 2.86$$

FIGURE 6.16 Finding $P(32 \le x \le 35)$ When $\mu = 33$ and $\sigma = .7$ by Using a Normal Table



which says that the mileage 35 is 2.86 standard deviations above the mean $\mu = 33$. Looking at Figure 6.16, we see that the area between 32 and 35 under the normal curve having mean $\mu = 33$ and standard deviation $\sigma = .7$ equals the area between -1.43 and 2.86 under the standard normal curve. This equals the area under the standard normal curve to the left of 2.86, which the normal table tells us is .9979, minus the area under the standard normal curve to the left of -1.43, which the normal table tells us is .0764. We summarize this result as follows:

$$P(32 \le x \le 35) = P\left(\frac{32 - 33}{.7} \le \frac{x - \mu}{\sigma} \le \frac{35 - 33}{.7}\right)$$
$$= P(-1.43 \le z \le 2.86) = .9979 - .0764 = .9215$$

This probability says that, if the competing automaker's claim is valid, then 92.15 percent of all of its midsize cars will get mileages between 32 mpg and 35 mpg.

Example 6.2 illustrates the general procedure for finding a probability about a normally distributed random variable x. We summarize this procedure in the following box:

Finding Normal Probabilities

1. Formulate the problem in terms of the random variable x.
2. Calculate relevant z values and restate the problem in terms of the standard normal random variable
$$z = \frac{x - \mu}{\sigma}$$
3. Find the required area under the standard normal curve by using the normal table.
4. Note that it is always useful to draw a picture illustrating the needed area before using the normal table.

EXAMPLE 6.3 The Car Mileage Case: Estimating Mileage

C

Recall from Example 6.2 that the competing automaker claims that the population of mileages of all its midsize cars is normally distributed with mean $\mu = 33$ and standard deviation $\sigma = .7$. Suppose that an independent testing agency randomly selects one of these cars and finds that it gets a mileage of 31.2 mpg when tested as prescribed by the EPA. Because the sample mileage of 31.2 mpg is *less than* the claimed mean $\mu = 33$, we have some evidence that contradicts the competing automaker's claim. To evaluate the strength of this evidence, we will calculate the probability that the mileage, x, of a randomly selected midsize car would be *less than or equal to* 31.2 if, in fact, the competing automaker's claim is true. To calculate $P(x \leq 31.2)$ under the assumption that the claim is true, we find the area to the left of 31.2 under the normal curve with mean $\mu = 33$ and standard deviation $\sigma = .7$ (see Figure 6.17). In order to use the normal table, we must find the z value corresponding to 31.2. This z value is

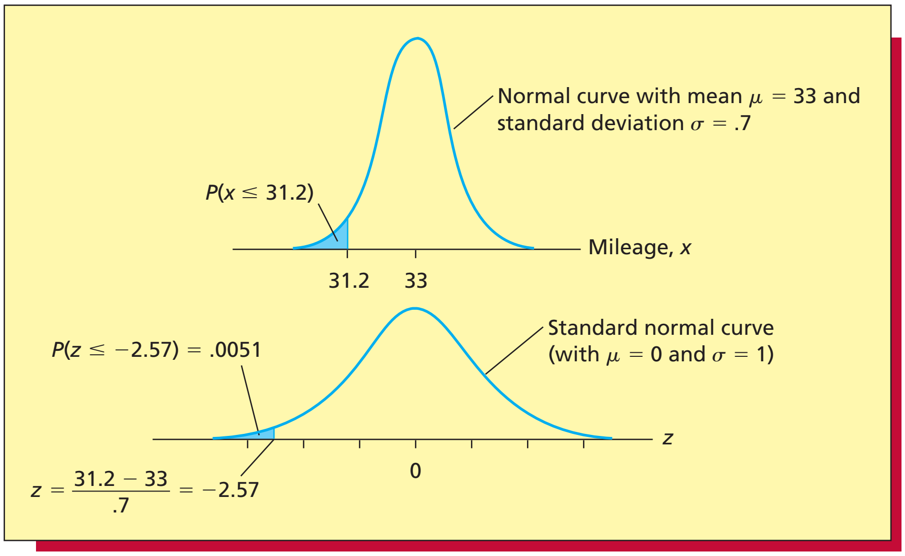
$$z = \frac{x - \mu}{\sigma} = \frac{31.2 - 33}{.7} = -2.57$$

which says that the mileage 31.2 is 2.57 standard deviations below the mean mileage $\mu = 33$. Looking at Figure 6.17, we see that the area to the left of 31.2 under the normal curve having mean $\mu = 33$ and standard deviation $\sigma = .7$ equals the area to the left of -2.57 under the standard normal curve. The normal table tells us that the area under the standard normal curve to the left of -2.57 is .0051, as shown in Figure 6.17. It follows that we can summarize our calculations as follows:

$$\begin{aligned} P(x \leq 31.2) &= P\left(\frac{x - \mu}{\sigma} \leq \frac{31.2 - 33}{.7}\right) \\ &= P(z \leq -2.57) = .0051 \end{aligned}$$

This probability says that, if the competing automaker's claim is valid, then only 51 in 10,000 cars would obtain a mileage of less than or equal to 31.2 mpg. Because it is very difficult to believe that a 51 in 10,000 chance has occurred, we have very strong evidence against the competing automaker's claim. It is probably true that μ is less than 33 and/or σ is greater than .7 and/or the population of all mileages is not normally distributed.

FIGURE 6.17 Finding $P(x \leq 31.2)$ When $\mu = 33$ and $\sigma = .7$ by Using a Normal Table



EXAMPLE 6.4 The Coffee Temperature Case: Meeting Customer Requirements C

Recall that marketing research done by a fast-food restaurant indicates that coffee tastes best if its temperature is between 153°(F) and 167°(F). The restaurant has sampled the coffee it serves and observed the 48 temperature readings in Table 1.10 on page 17. The temperature readings have a mean $\bar{x} = 159.3958$ and a standard deviation $s = 6.4238$ and are described by a bell-shaped histogram. Using $\bar{x}$ and s as point estimates of the mean μ and the standard deviation σ of the population of all possible coffee temperatures, we wish to calculate the probability that x, the temperature of a randomly selected cup of coffee, is outside the customer requirements for best-testing coffee (that is, less than 153° or greater than 167°). In order to compute the probability $P(x < 153 \text{ or } x > 167)$, we compute the z values

$$z = \frac{153 - 159.3958}{6.4238} = -1.00 \quad \text{and} \quad z = \frac{167 - 159.3958}{6.4238} = 1.18$$

Because the events $\{x < 153\}$ and $\{x > 167\}$ are mutually exclusive, we have

$$\begin{aligned} P(x < 153 \text{ or } x > 167) &= P(x < 153) + P(x > 167) \\ &= P(z < -1.00) + P(z > 1.18) \\ &= .1587 + .1190 = .2777 \end{aligned}$$

BI

This calculation is illustrated in Figure 6.18. The probability of .2777 implies that 27.77 percent of the coffee temperatures do not meet customer requirements and 72.23 percent of the coffee temperatures do meet these requirements. If management wishes a very high percentage of its coffee temperatures to meet customer requirements, the coffee-making process must be improved.

LO6-5 Find population values that correspond to specified normal distribution probabilities.

Finding a point on the horizontal axis under a normal curve In order to use many of the formulas given in later chapters, we must be able to find the z value so that the tail area to the right of z under the standard normal curve is a particular value. For instance, we might need to find the z value so that the tail area to the right of z under the standard normal curve is .025. This z value is denoted $z_{.025}$, and we illustrate $z_{.025}$ in Figure 6.19(a). We refer to $z_{.025}$ as **the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to .025.** It is easy to use the cumulative normal table to find such a point. For instance, in order to find $z_{.025}$, we note from Figure 6.19(b) that the area under the standard normal curve to the left of $z_{.025}$ equals .975. Remembering that areas under the standard normal curve to the left of z are

FIGURE 6.18 Finding $P(x < 153 \text{ or } x > 167)$ in the Coffee Temperature Case

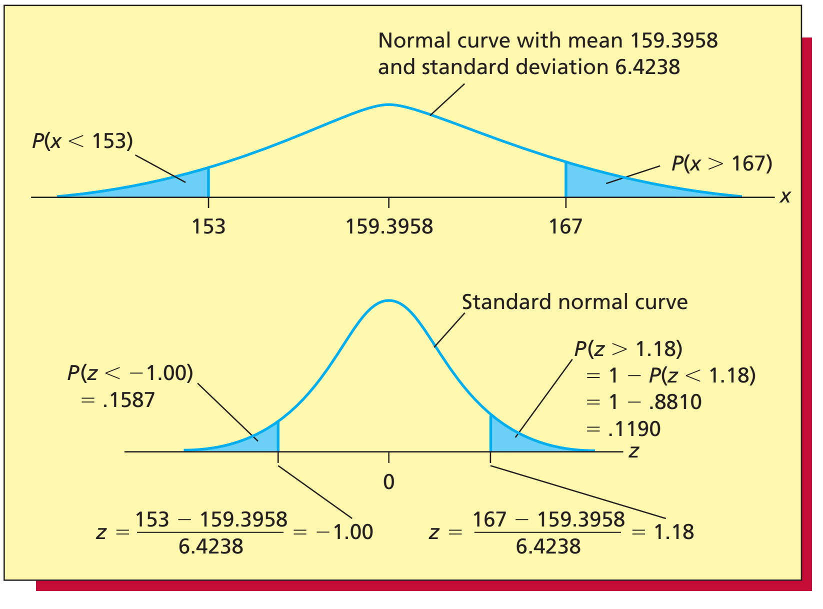
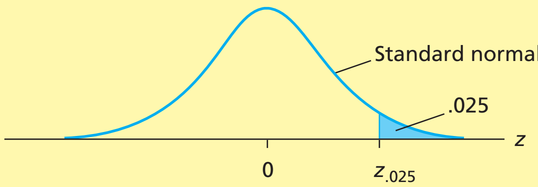
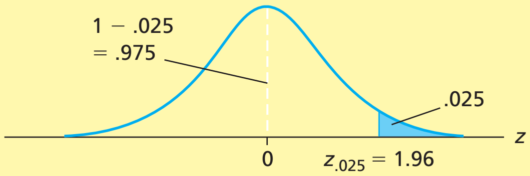


FIGURE 6.19 The Point $z_{.025} = 1.96$



(a) $z_{.025}$ is the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to .025



(b) Finding $z_{.025}$

the four-digit (or five-digit) numbers given in the body of Table 6.1, we scan the body of the table and find the area .9750. We have shaded this area in Table 6.1 on page 234, and we note that the area .9750 is in the row corresponding to a z of 1.9 and in the column headed by .06. It follows that the z value corresponding to .9750 is 1.96. Because the z value 1.96 gives an area under the standard normal curve to its left that equals .975, it also gives a right-hand tail area equal to .025. Therefore, $z_{.025} = 1.96$.

In general, **we let z_α denote the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to α.** With this definition in mind, we consider the following example.

EXAMPLE 6.5 The DVD Case: Managing Inventory

A large discount store sells 50 packs of HX-150 blank DVDs and receives a shipment every Monday. Historical sales records indicate that the weekly demand, x, for these 50 packs is normally distributed with a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$. How many 50 packs should be stocked at the beginning of a week so that there is only a 5 percent chance that the store will run short during the week?

If we let st equal the number of 50 packs that will be stocked, then st must be chosen to allow only a .05 probability that weekly demand, x, will exceed st. That is, st must be chosen so that

$$P(x > st) = .05$$

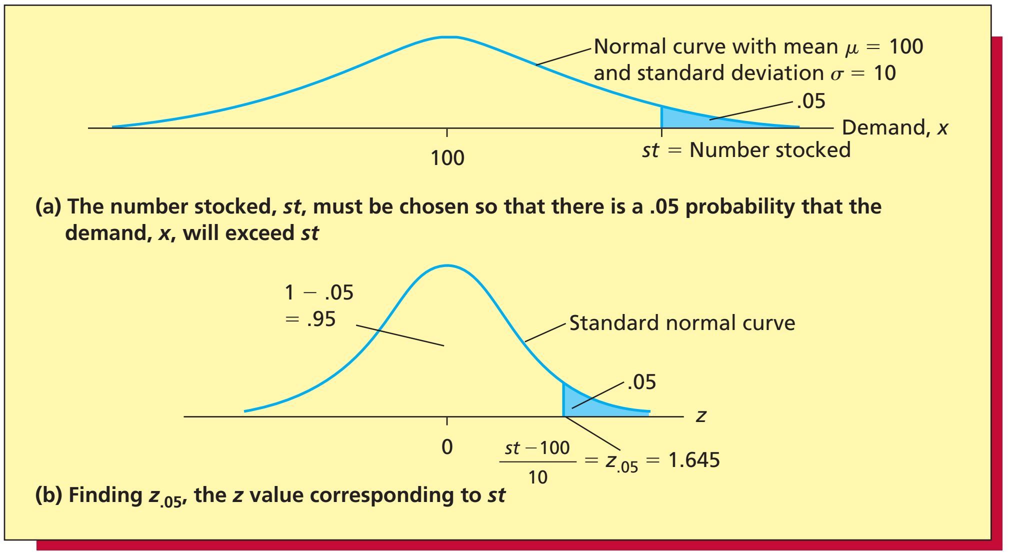
Figure 6.20(a) on the next page shows that the number stocked, st, is located under the right-hand tail of the normal curve having mean $\mu = 100$ and standard deviation $\sigma = 10$. In order to find st, we need to determine how many standard deviations st must be above the mean in order to give a right-hand tail area that is equal to .05.

The z value corresponding to st is

$$z = \frac{st - \mu}{\sigma} = \frac{st - 100}{10}$$

and this z value is the number of standard deviations that st is from μ. This z value is illustrated in Figure 6.20(b), and it is the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to .05. That is, the z value corresponding to st is $z_{.05}$. Since the area under the standard normal curve to the left of $z_{.05}$ is $1 - .05 = .95$—see Figure 6.20(b)—we look for .95 in the body of the normal table. In Table 6.1, we see that the areas closest to .95 are .9495, which has a corresponding z value of 1.64, and .9505, which has a corresponding z value of 1.65. Although it

FIGURE 6.20 Finding the Number of 50 Packs of DVDs Stocked, st, so That $P(x > st) = .05$ When $\mu = 100$ and $\sigma = 10$



would probably be sufficient to use either of these z values, we will (because it is easy to do so) interpolate halfway between them and assume that $z_{.05}$ equals 1.645. To find st, we solve the equation

$$\frac{st - 100}{10} = 1.645$$

for st. Doing this yields

$$st - 100 = 1.645(10)$$

or

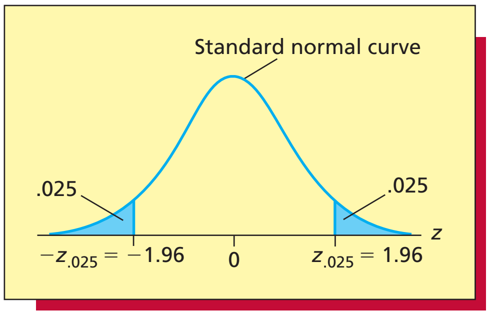
$$st = 100 + 1.645(10) = 116.45$$

BI

This last equation says that st is 1.645 standard deviations ($\sigma = 10$) above the mean ($\mu = 100$). Rounding $st = 116.45$ up so that the store's chances of running short will be *no more* than 5 percent, the store should stock 117 of the 50 packs at the beginning of each week.

Sometimes we need to find the point on the horizontal axis under the standard normal curve that gives a particular **left-hand tail area** (say, for instance, an area of .025). Looking at Figure 6.21, it is easy to see that, if, for instance, we want a left-hand tail area of .025, the needed z value is $-z_{.025}$, where $z_{.025}$ gives a right-hand tail area equal to .025. To find $-z_{.025}$, we look for .025 in the body of the normal table and find that the z value corresponding to .025 is -1.96. Therefore, $-z_{.025} = -1.96$. In general, **$-z_\alpha$ is the point on the horizontal axis under the standard normal curve that gives a left-hand tail area equal to α.**

FIGURE 6.21 The z Value $-z_{.025} = -1.96$ Gives a Left-Hand Tail Area of .025 under the Standard Normal Curve



EXAMPLE 6.6 Setting A Guarantee Period

Extensive testing indicates that the lifetime of the Everlast automobile battery is normally distributed with a mean of $\mu = 60$ months and a standard deviation of $\sigma = 6$ months. The Everlast's manufacturer has decided to offer a free replacement battery to any purchaser whose Everlast battery does not last at least as long as the minimum lifetime specified in its guarantee. How can the manufacturer establish the guarantee period so that only 1 percent of the batteries will need to be replaced free of charge?

If the battery will be guaranteed to last l months, l must be chosen to allow only a .01 probability that the lifetime, x, of an Everlast battery will be less than l. That is, we must choose l so that

$$P(x < l) = .01$$

Figure 6.22(a) shows that the guarantee period, l, is located under the left-hand tail of the normal curve having mean $\mu = 60$ and standard deviation $\sigma = 6$. In order to find l, we need to determine how many standard deviations l must be below the mean in order to give a left-hand tail area that equals .01. The z value corresponding to l is

$$z = \frac{l - \mu}{\sigma} = \frac{l - 60}{6}$$

and this z value is the number of standard deviations that l is from μ. This z value is illustrated in Figure 6.22(b), and it is the point on the horizontal axis under the standard normal curve that gives a left-hand tail area equal to .01. That is, the z value corresponding to l is $-z_{.01}$. To find $-z_{.01}$, we look for .01 in the body of the normal table. Doing this, we see that the area closest to .01 is .0099, which has a corresponding z value of -2.33. Therefore, $-z_{.01}$ is (roughly) -2.33. To find l, we solve the equation

$$\frac{l - 60}{6} = -2.33$$

for l. Doing this yields

$$l - 60 = -2.33(6)$$

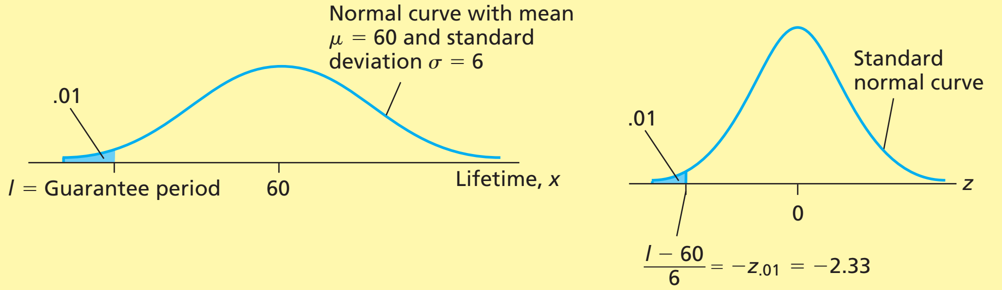
or

$$l = 60 - 2.33(6) = 46.02$$

Note that this last equation says that l is 2.33 standard deviations ($\sigma = 6$) below the mean ($\mu = 60$). Rounding $l = 46.02$ down so that *no more* than 1 percent of the batteries will need to be replaced free of charge, it seems reasonable to guarantee the Everlast battery to last 46 months.

BI

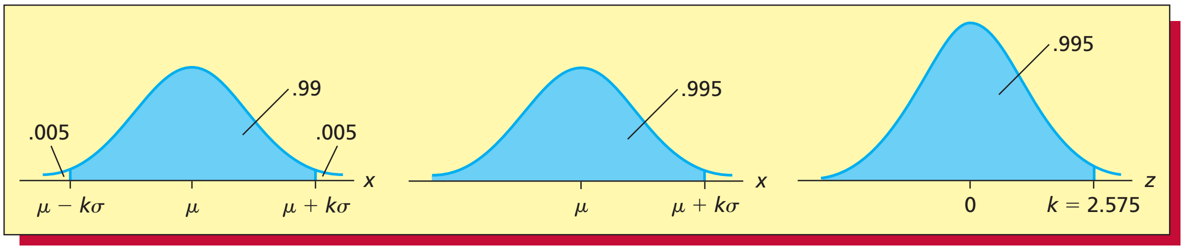
FIGURE 6.22 Finding the Guarantee Period, l, so That $P(x < l) = .01$ When $\mu = 60$ and $\sigma = 6$



(a) The guarantee period, l, must be chosen so that there is a .01 probability that the lifetime, x, will be less than l

(b) Finding $-z_{.01}$, the z value corresponding to l

FIGURE 6.23 Finding a Tolerance Interval $[\mu \pm k\sigma]$ That Contains 99 Percent of the Measurements in a Normally Distributed Population



Earlier in this section we saw that the intervals $[\mu \pm \sigma]$, $[\mu \pm 2\sigma]$, and $[\mu \pm 3\sigma]$ are **tolerance intervals** containing, respectively, 68.26 percent, 95.44 percent, and 99.73 percent of the measurements in a normally distributed population having mean μ and standard deviation σ. In the following example we demonstrate how to use the normal table to find the value k so that the interval $[\mu \pm k\sigma]$ contains any desired percentage of the measurements in a normally distributed population.

EXAMPLE 6.7 The Car Mileage Case: Estimating Mileage

Consider computing a tolerance interval $[\mu \pm k\sigma]$ that contains 99 percent of the measurements in a normally distributed population having mean μ and standard deviation σ. As illustrated in Figure 6.23, we must find the value k so that the area under the normal curve having mean μ and standard deviation σ between $(\mu - k\sigma)$ and $(\mu + k\sigma)$ is .99. Because the total area under this normal curve is 1, the area under the normal curve that is not between $(\mu - k\sigma)$ and $(\mu + k\sigma)$ is $1 - .99 = .01$. This implies, as illustrated in Figure 6.23, that the area under the normal curve to the left of $(\mu - k\sigma)$ is $.01/2 = .005$, and the area under the normal curve to the right of $(\mu + k\sigma)$ is also $.01/2 = .005$. This further implies, as illustrated in Figure 6.23, that the area under the normal curve to the left of $(\mu + k\sigma)$ is .995. Because the z value corresponding to a value of x tells us how many standard deviations x is from μ, the z value corresponding to $(\mu + k\sigma)$ is obviously k. It follows that k is the point on the horizontal axis under the standard normal curve so that the area to the left of k is .995. Looking up .995 in the body of the normal table, we find that the values closest to .995 are .9949, which has a corresponding z value of 2.57, and .9951, which has a corresponding z value of 2.58. Although it would be sufficient to use either of these z values, we will interpolate halfway between them, and we will assume that k equals 2.575. It follows that the interval $[\mu \pm 2.575\sigma]$ contains 99 percent of the measurements in a normally distributed population having mean μ and standard deviation σ. For example, recall that the histogram of the sample of 50 midsize car mileages in Figure 3.15 (page 115) suggests that the population of all midsize car mileages is normally distributed. Because the sample mean and the sample standard deviation are $\bar{x} = 31.56$ and $s = .7977$, we estimate that 99 percent of the new midsize cars will obtain mileages in the interval $[\bar{x} \pm 2.575s] = [31.56 \pm 2.575(.7977)] = [29.5, 33.6]$—that is, will get mileages between 29.5 mpg and 33.6 mpg.

Whenever we use a normal table to find a z point corresponding to a particular normal curve area, we will use the *halfway interpolation* procedure illustrated in Examples 6.5 and 6.7 if the area we are looking for is exactly halfway between two areas in the table. Otherwise, as illustrated in Example 6.6, we will use the z value corresponding to the area in the table that is closest to the desired area.

Exercises for Section 6.3

CONCEPTS

connect

6.16 List five important properties of the normal probability curve.

6.17 Explain what the mean, μ, tells us about a normal curve, and explain what the standard deviation, σ, tells us about a normal curve.

6.18 If the random variable x is normally distributed, what percentage of all possible observed values of x will be

a Within one standard deviation of the mean?
b Within two standard deviations of the mean?
c Within three standard deviations of the mean?

6.19 Explain how to compute the z value corresponding to a value of a normally distributed random variable. What does the z value tell us about the value of the random variable?

6.20 Explain how x relates to the mean μ if the z value corresponding to x

a Equals zero.
b Is positive.
c Is negative.

6.21 Why do we compute z values when using the normal table? Explain.

METHODS AND APPLICATIONS

6.22 In each case, sketch the two specified normal curves on the same set of axes:

a A normal curve with $\mu = 20$ and $\sigma = 3$, and a normal curve with $\mu = 20$ and $\sigma = 6$.
b A normal curve with $\mu = 20$ and $\sigma = 3$, and a normal curve with $\mu = 30$ and $\sigma = 3$.
c A normal curve with $\mu = 100$ and $\sigma = 10$, and a normal curve with $\mu = 200$ and $\sigma = 20$.

6.23 Let x be a normally distributed random variable having mean $\mu = 30$ and standard deviation $\sigma = 5$. Find the z value for each of the following observed values of x:

a $x = 25$
b $x = 15$
c $x = 30$
d $x = 40$
e $x = 50$

In each case, explain what the z value tells us about how the observed value of x compares to the mean, μ.

6.24 If the random variable z has a standard normal distribution, sketch and find each of the following probabilities:

a $P(0 \le z \le 1.5)$
b $P(z \ge 2)$
c $P(z \le 1.5)$
d $P(z \ge -1)$
e $P(z \le -3)$
f $P(-1 \le z \le 1)$
g $P(-2.5 \le z \le .5)$
h $P(1.5 \le z \le 2)$
i $P(-2 \le z \le -.5)$

6.25 Suppose that the random variable z has a standard normal distribution. Sketch each of the following z points, and use the normal table to find each z point.

a $z_{.01}$
b $z_{.05}$
c $z_{.02}$
d $-z_{.01}$
e $-z_{.05}$
f $-z_{.10}$

6.26 Suppose that the random variable x is normally distributed with mean $\mu = 1{,}000$ and standard deviation $\sigma = 100$. Sketch and find each of the following probabilities:

a $P(1{,}000 \le x \le 1{,}200)$
b $P(x > 1{,}257)$
c $P(x < 1{,}035)$
d $P(857 \le x \le 1{,}183)$
e $P(x \le 700)$
f $P(812 \le x \le 913)$
g $P(x > 891)$
h $P(1{,}050 \le x \le 1{,}250)$

6.27 Suppose that the random variable x is normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$. For each of the following, use the normal table to find the needed value k. In each case, draw a sketch.

a $P(x \ge k) = .025$
b $P(x \ge k) = .05$
c $P(x < k) = .025$
d $P(x \le k) = .015$
e $P(x < k) = .985$
f $P(x > k) = .95$
g $P(x \le k) = .975$
h $P(x \ge k) = .0228$
i $P(x > k) = .9772$

6.28 Stanford–Binet IQ Test scores are normally distributed with a mean score of 100 and a standard deviation of 16.

a Sketch the distribution of Stanford–Binet IQ test scores.
b Write the equation that gives the z score corresponding to a Stanford–Binet IQ test score. Sketch the distribution of such z scores.
c Find the probability that a randomly selected person has an IQ test score

(1) Over 140.
(2) Under 88.
(3) Between 72 and 128.
(4) Within 1.5 standard deviations of the mean.

d Suppose you take the Stanford–Binet IQ Test and receive a score of 136. What percentage of people would receive a score higher than yours?

6.29 Weekly demand at a grocery store for a brand of breakfast cereal is normally distributed with a mean of 800 boxes and a standard deviation of 75 boxes.

a What is the probability that weekly demand is

(1) 959 boxes or less?

(2) More than 1,004 boxes?

(3) Less than 650 boxes or greater than 950 boxes?

b The store orders cereal from a distributor weekly. How many boxes should the store order for a week to have only a 2.5 percent chance of running short of this brand of cereal during the week?

6.30 The lifetimes of a particular brand of DVD player are normally distributed with a mean of eight years and a standard deviation of six months. Find each of the following probabilities where x denotes the lifetime in years. In each case, sketch the probability.

a $P(7 \leq x \leq 9)$ **e** $P(x \leq 7)$

b $P(8.5 \leq x \leq 9.5)$ **f** $P(x \geq 7)$

c $P(6.5 \leq x \leq 7.5)$ **g** $P(x \leq 10)$

d $P(x \geq 8)$ **h** $P(x > 10)$

6.31 United Motors claims that one of its cars, the Starbird 300, gets city driving mileages that are normally distributed with a mean of 30 mpg and a standard deviation of 1 mpg. Let x denote the city driving mileage of a randomly selected Starbird 300.

a Assuming that United Motors' claim is correct, find $P(x \leq 27)$.

b If you purchase (randomly select) a Starbird 300 and your car gets 27 mpg in city driving, what do you think of United Motors' claim? Explain your answer.

6.32 An investment broker reports that the yearly returns on common stocks are approximately normally distributed with a mean return of 12.4 percent and a standard deviation of 20.6 percent. On the other hand, the firm reports that the yearly returns on tax-free municipal bonds are approximately normally distributed with a mean return of 5.2 percent and a standard deviation of 8.6 percent. Find the probability that a randomly selected

a Common stock will give a positive yearly return.

b Tax-free municipal bond will give a positive yearly return.

c Common stock will give more than a 10 percent return.

d Tax-free municipal bond will give more than a 10 percent return.

e Common stock will give a loss of at least 10 percent.

f Tax-free municipal bond will give a loss of at least 10 percent.

6.33 A filling process is supposed to fill jars with 16 ounces of grape jelly. Specifications state that each jar must contain between 15.95 ounces and 16.05 ounces. A jar is selected from the process every half hour until a sample of 100 jars is obtained. When the fills of the jars are measured, it is found that $\bar{x} = 16.0024$ and $s = .02454$. Using $\bar{x}$ and s as point estimates of μ and σ, estimate the probability that a randomly selected jar will have a fill, x, that is out of specification. Assume that the process is in control and that the population of all jar fills is normally distributed.

6.34 A tire company has developed a new type of steel-belted radial tire. Extensive testing indicates the population of mileages obtained by all tires of this new type is normally distributed with a mean of 40,000 miles and a standard deviation of 4,000 miles. The company wishes to offer a guarantee providing a discount on a new set of tires if the original tires purchased do not exceed the mileage stated in the guarantee. What should the guaranteed mileage be if the tire company desires that no more than 2 percent of the tires will fail to meet the guaranteed mileage?

6.35 Recall from Exercise 6.32 that yearly returns on common stocks are normally distributed with a mean of 12.4 percent and a standard deviation of 20.6 percent.

a What percentage of yearly returns are at or below the 10th percentile of the distribution of yearly returns? What percentage are at or above the 10th percentile? Find the 10th percentile of the distribution of yearly returns.

b Find the first quartile, Q_1, and the third quartile, Q_3, of the distribution of yearly returns.

6.36 Two students take a college entrance exam known to have a normal distribution of scores. The students receive raw scores of 63 and 93, which correspond to z scores (often called the standardized scores) of -1 and 1.5, respectively. Find the mean and standard deviation of the distribution of raw exam scores.

6.37 THE TRASH BAG CASE DS TrashBag

Suppose that a population of measurements is normally distributed with mean μ and standard deviation σ.

a Write an expression (involving μ and σ) for a tolerance interval containing 98 percent of all the population measurements.

b Estimate a tolerance interval containing 98 percent of all the trash bag breaking strengths by using the fact that a random sample of 40 breaking strengths has a mean of $\bar{x} = 50.575$ and a standard deviation of $s = 1.6438$.

6.38 Consider the situation of Exercise 6.32.

a Use the investment broker's report to estimate the maximum yearly return that might be obtained by investing in tax-free municipal bonds.

b Find the probability that the yearly return obtained by investing in common stocks will be higher than the maximum yearly return that might be obtained by investing in tax-free municipal bonds.

6.39 In the book *Advanced Managerial Accounting,* Robert P. Magee discusses monitoring cost variances. A *cost variance* is the difference between a budgeted cost and an actual cost. Magee describes the following situation:

> Michael Bitner has responsibility for control of two manufacturing processes. Every week he receives a cost variance report for each of the two processes, broken down by labor costs, materials costs, and so on. One of the two processes, which we'll call process *A*, involves a stable, easily controlled production process with a little fluctuation in variances. Process *B* involves more random events: the equipment is more sensitive and prone to breakdown, the raw material prices fluctuate more, and so on.
>
> "It seems like I'm spending more of my time with process *B* than with process *A*," says Michael Bitner. "Yet I know that the probability of an inefficiency developing and the expected costs of inefficiencies are the same for the two processes. It's just the magnitude of random fluctuations that differs between the two, as you can see in the information below.
>
> "At present, I investigate variances if they exceed \$2,500, regardless of whether it was process *A* or *B.* I suspect that such a policy is not the most efficient. I should probably set a higher limit for process *B.*"

The means and standard deviations of the cost variances of processes *A* and *B*, when these processes are in control, are as follows:

	Process *A*	Process *B*
Mean cost variance (in control)	\$ 0	\$ 0
Standard deviation of cost variance (in control)	\$5,000	\$10,000

Furthermore, the means and standard deviations of the cost variances of processes *A* and *B*, when these processes are out of control, are as follows:

	Process *A*	Process *B*
Mean cost variance (out of control)	\$7,500	\$ 7,500
Standard deviation of cost variance (out of control)	\$5,000	\$10,000

a Recall that the current policy is to investigate a cost variance if it exceeds \$2,500 for either process. Assume that cost variances are normally distributed and that both Process *A* and Process *B* cost variances are in control. Find the probability that a cost variance for Process *A* will be investigated. Find the probability that a cost variance for Process *B* will be investigated. Which in-control process will be investigated more often?

b Assume that cost variances are normally distributed and that both Process *A* and Process *B* cost variances are out of control. Find the probability that a cost variance for Process *A* will be investigated. Find the probability that a cost variance for Process *B* will be investigated. Which out-of-control process will be investigated more often?

c If both Processes *A* and *B* are almost always in control, which process will be investigated more often?

d Suppose that we wish to reduce the probability that Process *B* will be investigated (when it is in control) to .3085. What cost variance investigation policy should be used? That is, how large a cost variance should trigger an investigation? Using this new policy, what is the probability that an out-of-control cost variance for Process *B* will be investigated?

6.40 Suppose that yearly health care expenses for a family of four are normally distributed with a mean expense equal to \$3,000 and a standard deviation of \$500. An insurance company has decided to offer a health insurance premium reduction if a policyholder's health care expenses do not exceed a specified dollar amount. What dollar amount should be established if the insurance company wants families having the lowest 33 percent of yearly health care expenses to be eligible for the premium reduction?

6.41 Suppose that the 33rd percentile of a normal distribution is equal to 656 and that the 97.5th percentile of this normal distribution is 896. Find the mean μ and the standard deviation σ of the normal distribution. Hint: Sketch these percentiles.

LO6-6 Use the normal distribution to approximate binomial probabilities (Optional).

6.4 Approximating the Binomial Distribution by Using the Normal Distribution (Optional)

Figure 6.24 illustrates several binomial distributions. In general, we can see that as n gets larger and as p gets closer to .5, the graph of a binomial distribution tends to have the symmetrical, bell-shaped appearance of a normal curve. It follows that, under conditions given in the following box, we can approximate the binomial distribution by using a normal distribution.

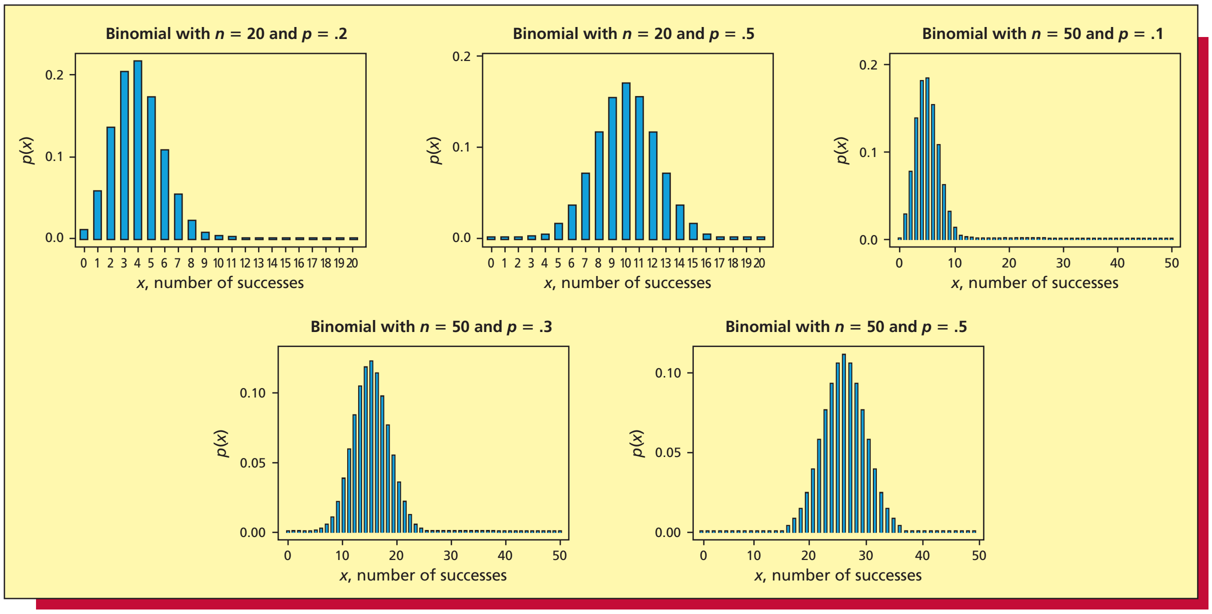
The Normal Approximation of the Binomial Distribution

Consider a binomial random variable *x*, where n is the number of trials performed and p is the probability of success on each trial. If n and p have values so that $np \geq 5$ and $n(1 - p) \geq 5$, then x is approximately normally distributed with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$, where $q = 1 - p$.

This approximation is often useful because binomial tables for large values of n are often unavailable. The conditions $np \geq 5$ and $n(1 - p) \geq 5$ must be met in order for the approximation to be appropriate. Note that if p is near 0 or near 1, then n must be larger for a good approximation, while if p is near .5, then n need not be as large.[1]

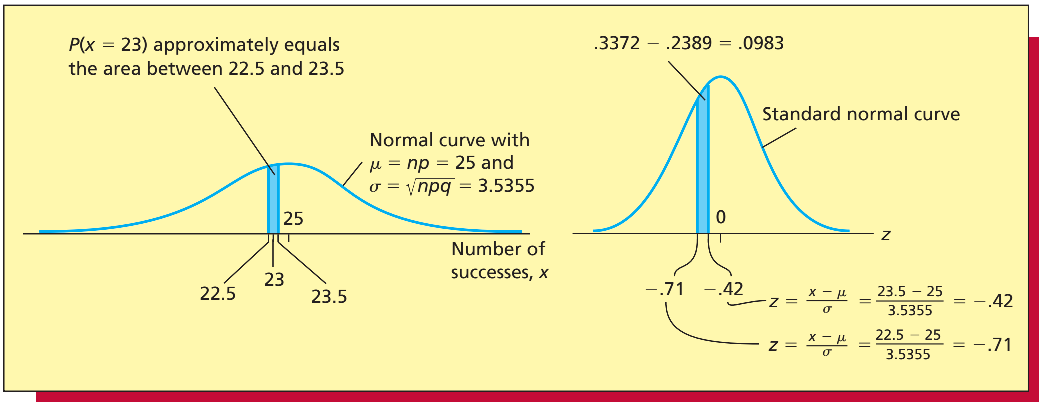
When we say that we can approximate the binomial distribution by using a normal distribution, we are saying that we can compute binomial probabilities by finding corresponding areas under a normal curve (rather than by using the binomial formula). We illustrate how to do this in the following example.

FIGURE 6.24 Several Binomial Distributions



[1]As an alternative to the rule that both np and $n(1 - p)$ must be at least 5, some statisticians suggest using the more conservative rule that both np and $n(1 - p)$ must be at least 10.

FIGURE 6.25 Approximating the Binomial Probability $P(x = 23)$ by Using the Normal Curve When $\mu = np = 25$ and $\sigma = \sqrt{npq} = 3.5355$



EXAMPLE 6.8 The Continuity Correction

Consider the binomial random variable x with $n = 50$ trials and probability of success $p = .5$. This binomial distribution is one of those illustrated in Figure 6.24. Suppose we want to use the normal approximation to this binomial distribution to compute the probability of 23 successes in the 50 trials. That is, we wish to compute $P(x = 23)$. Because $np = (50)(.5) = 25$ is at least 5, and $n(1 - p) = 50(1 - .5) = 25$ is also at least 5, we can appropriately use the approximation. Moreover, we can approximate the binomial distribution of x by using a normal distribution with mean $\mu = np = 50(.5) = 25$ and standard deviation $\sigma = \sqrt{npq} = \sqrt{50(.5)(1 - .5)} = 3.5355$.

In order to compute the needed probability, we must make a **continuity correction.** This is because a discrete distribution (the binomial) is being approximated by a continuous distribution (the normal). Because there is no area under a normal curve at the single point $x = 23$, we must assign an area under the normal curve to the binomial outcome $x = 23$. It is logical to assign the area corresponding to the interval from 22.5 to 23.5 to the integer outcome $x = 23$. That is, the area under the normal curve corresponding to all values within .5 unit of the integer outcome $x = 23$ is assigned to the value $x = 23$. So we approximate the binomial probability $P(x = 23)$ by calculating the normal curve area $P(22.5 \leq x \leq 23.5)$. This area is illustrated in Figure 6.25. Calculating the z values

$$z = \frac{22.5 - 25}{3.5355} = -.71 \qquad \text{and} \qquad z = \frac{23.5 - 25}{3.5355} = -.42$$

we find that $P(22.5 \leq x \leq 23.5) = P(-.71 \leq z \leq -.42) = .3372 - .2389 = .0983$. Therefore, we estimate that the binomial probability $P(x = 23)$ is .0983.

Making the proper continuity correction can sometimes be tricky. A good way to approach this is to list the numbers of successes that are included in the event for which the binomial probability is being calculated. Then assign the appropriate area under the normal curve to each number of successes in the list. Putting these areas together gives the normal curve area that must be calculated. For example, again consider the binomial random variable x with $n = 50$ and $p = .5$. If we wish to find $P(27 \leq x \leq 29)$, then the event $27 \leq x \leq 29$ includes 27, 28, and 29 successes. Because we assign the areas under the normal curve corresponding to the intervals [26.5, 27.5], [27.5, 28.5], and [28.5, 29.5] to the values 27, 28, and 29, respectively, then the area to be found under the normal curve is $P(26.5 \leq x \leq 29.5)$. Table 6.2 on the next page gives several other examples.

TABLE 6.2 Several Examples of the Continuity Correction ($n = 50$)

Binomial Probability	Numbers of Successes Included in Event	Normal Curve Area (with Continuity Correction)
$P(25 < x \leq 30)$	26, 27, 28, 29, 30	$P(25.5 \leq x \leq 30.5)$
$P(x \leq 27)$	0, 1, 2, . . . , 26, 27	$P(x \leq 27.5)$
$P(x > 30)$	31, 32, 33, . . . , 50	$P(x \geq 30.5)$
$P(27 < x < 31)$	28, 29, 30	$P(27.5 \leq x \leq 30.5)$

EXAMPLE 6.9 The Cheese Spread Case: Improving Profitability

C

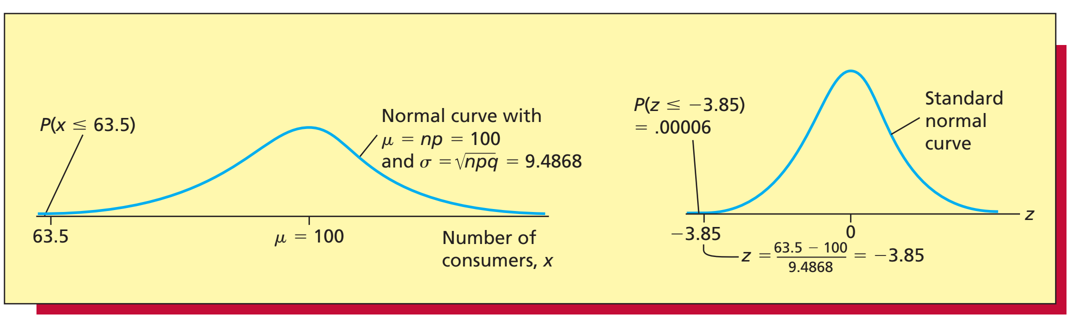
A food processing company markets a soft cheese spread that is sold in a plastic container with an "easy pour" spout. Although this spout works extremely well and is popular with consumers, it is expensive to produce. Because of the spout's high cost, the company has developed a new, less expensive spout. While the new, cheaper spout may alienate some purchasers, a company study shows that its introduction will increase profits if fewer than 10 percent of the cheese spread's current purchasers are lost. That is, if we let p be the true proportion of all current purchasers who would stop buying the cheese spread if the new spout were used, profits will increase as long as p is less than .10.

Suppose that (after trying the new spout) 63 of 1,000 randomly selected purchasers say that they would stop buying the cheese spread if the new spout were used. To assess whether p is less than .10, we will assume for the sake of argument that p equals .10, and we will use the sample information to weigh the evidence against this assumption and in favor of the conclusion that p is less than .10. Let the random variable x represent the number of the 1,000 purchasers who say they would stop buying the cheese spread. Assuming that p equals .10, then x is a binomial random variable with $n = 1{,}000$ and $p = .10$. Because the sample result of 63 is less than $\mu = np = 1{,}000(.1) = 100$, the expected value of x when p equals .10, we have some evidence to contradict the assumption that p equals .10. To evaluate the strength of this evidence, we calculate the probability that *63 or fewer* of the 1,000 randomly selected purchasers would say that they would stop buying the cheese spread if the new spout were used if, in fact, p equals .10.

Because both $np = 1{,}000(.10) = 100$ and $n(1 - p) = 1{,}000(1 - .10) = 900$ are at least 5, we can use the normal approximation to the binomial distribution to compute the needed probability. The appropriate normal curve has mean $\mu = np = 1{,}000(.10) = 100$ and standard deviation $\sigma = \sqrt{npq} = \sqrt{1{,}000(.10)(1 - .10)} = 9.4868$. In order to make the continuity correction, we note that the discrete value $x = 63$ is assigned the area under the normal curve corresponding to the interval from 62.5 to 63.5. It follows that the binomial probability $P(x \leq 63)$ is approximated by the normal probability $P(x \leq 63.5)$. This is illustrated in Figure 6.26. Calculating the z value for 63.5 to be

$$z = \frac{63.5 - 100}{9.4868} = -3.85$$

FIGURE 6.26 Approximating the Binomial Probability $P(x \leq 63)$ by Using the Normal Curve When $\mu = np = 100$ and $\sigma = \sqrt{npq} = 9.4868$



we find that

$$P(x \leq 63.5) = P(z \leq -3.85)$$

Using the normal table, we find that the area under the standard normal curve to the left of -3.85 is .00006. This says that, if p equals .10, then in only 6 in 100,000 of all possible random samples of 1,000 purchasers would 63 or fewer say they would stop buying the cheese spread if the new spout were used. Because it is very difficult to believe that such a small chance (a .00006 chance) has occurred, we have very strong evidence that p does not equal .10 and is, in fact, less than .10. Therefore, it seems that using the new spout will be profitable.

Exercises for Section 6.4

CONCEPTS

connect

6.42 Explain why it might be convenient to approximate binomial probabilities by using areas under an appropriate normal curve.

6.43 Under what condition may we use the normal approximation to the binomial distribution?

6.44 Explain how we make a continuity correction. Why is a continuity correction needed when we approximate a binomial distribution by a normal distribution?

METHODS AND APPLICATIONS

6.45 Suppose that x has a binomial distribution with $n = 200$ and $p = .4$.

a Show that the normal approximation to the binomial can appropriately be used to calculate probabilities about x.

b Make continuity corrections for each of the following, and then use the normal approximation to the binomial to find each probability:

(1) $P(x = 80)$
(2) $P(x \leq 95)$
(3) $P(x < 65)$
(4) $P(x \geq 100)$
(5) $P(x > 100)$

6.46 Repeat Exercise 6.45 with $n = 200$ and $p = .5$.

6.47 An advertising agency conducted an ad campaign aimed at making consumers in an Eastern state aware of a new product. Upon completion of the campaign, the agency claimed that 20 percent of consumers in the state had become aware of the product. The product's distributor surveyed 1,000 consumers in the state and found that 150 were aware of the product.

a Assuming that the ad agency's claim is true:

(1) Verify that we may use the normal approximation to the binomial.
(2) Calculate the mean, μ, and the standard deviation, σ, we should use in the normal approximation.
(3) Find the probability that 150 or fewer consumers in a random sample of 1,000 consumers would be aware of the product.

b Should the distributor believe the ad agency's claim? Explain.

6.48 In order to gain additional information about respondents, some marketing researchers have used ultraviolet ink to precode questionnaires that promise confidentiality to respondents. Of 205 randomly selected marketing researchers who participated in an actual survey, 117 said that they disapprove of this practice. Suppose that, before the survey was taken, a marketing manager claimed that at least 65 percent of all marketing researchers would disapprove of the practice.

a Assuming that the manager's claim is correct, calculate the probability that 117 or fewer of 205 randomly selected marketing researchers would disapprove of the practice. Use the normal approximation to the binomial.

b Based on your result of part *a*, do you believe the marketing manager's claim? Explain.

6.49 When a store uses electronic article surveillance (EAS) to combat shoplifting, it places a small sensor on each item of merchandise. When an item is legitimately purchased, the sales clerk is supposed to remove the sensor to prevent an alarm from sounding as the customer exits the store. In an actual survey of 250 consumers, 40 said that if they were to set off an EAS alarm because store personnel (mistakenly) failed to deactivate merchandise leaving the store, they would never shop at that store again. A company marketing the alarm system claimed that no more than 5 percent of all consumers would say that they would never shop at that store again if they were subjected to a false alarm.

a Assuming that the company's claim is valid, use the normal approximation to the binomial to calculate the probability that at least 40 of the 250 randomly selected consumers would say that they would never shop at that store again if they were subjected to a false alarm.

b Do you believe the company's claim based on your answer to part *a*? Explain.

6.50 A department store will place a sale item in a special display for a one-day sale. Previous experience suggests that 20 percent of all customers who pass such a special display will purchase the item. If 2,000 customers will pass the display on the day of the sale, and if a one-item-per-customer limit is placed on the sale item, how many units of the sale item should the store stock in order to have at most a 1 percent chance of running short of the item on the day of the sale? Assume here that customers make independent purchase decisions.

6.5 The Exponential Distribution (Optional)

LO6-7 Use the exponential distribution to compute probabilities (Optional).

In Example 5.11 (pages 208–210), we considered an air traffic control center where controllers occasionally misdirect pilots onto flight paths dangerously close to those of other aircraft. We found that the number of these controller errors in a given time period has a Poisson distribution and that the control center is averaging 20.8 errors per year. However, rather than focusing on the number of errors occurring in a given time period, we could study the time elapsing between successive errors. If we let x denote the number of weeks elapsing between successive errors, then x is a continuous random variable that is described by what is called the *exponential distribution.* Moreover, because the control center is averaging 20.8 errors per year, the center is averaging a mean, denoted λ, of $20.8/52 = .4$ error per week and thus a mean of $52/20.8 = 2.5$ (that is, $1/\lambda = 1/.4 = 2.5$) weeks between successive errors.

In general, if the number of events occurring per unit of time or space (for example, the number of controller errors per week or the number of imperfections per square yard of cloth) has a Poisson distribution with mean λ, then the number of units, x, of time or space between successive events has an *exponential distribution* with mean $1/\lambda$. The equation of the probability curve describing the exponential distribution is given in the following formula box.

The Exponential Distribution

If x is described by an exponential distribution with mean $1/\lambda$, then the equation of the probability curve describing x is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Using this probability curve, it can be shown that:

$$P(a \leq x \leq b) = e^{-\lambda a} - e^{-\lambda b}$$

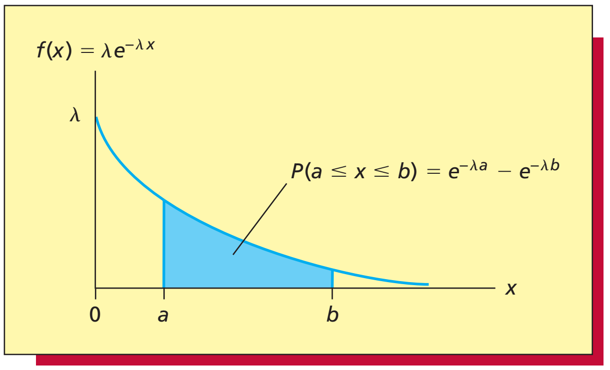
In particular, because $e^0 = 1$ and $e^{-\infty} = 0$, this implies that

$$P(x \leq b) = 1 - e^{-\lambda b} \quad \text{and} \quad P(x \geq a) = e^{-\lambda a}$$

Furthermore, both the mean and the standard deviation of the population of all possible observed values of a random variable x that has an exponential distribution are equal to $1/\lambda$. That is, $\mu_x = \sigma_x = 1/\lambda$.

The graph of the equation describing the exponential distribution and the probability $P(a \leq x \leq b)$ where x is described by this exponential distribution are illustrated in Figure 6.27.

FIGURE 6.27 A Graph of the Exponential Distribution $f(x) = \lambda e^{-\lambda x}$



We illustrate the use of the exponential distribution in the following examples.

EXAMPLE 6.10 The Air Safety Case: Traffic Control Errors

We have seen in the air traffic control example that the control center is averaging $\lambda = .4$ error per week and $1/\lambda = 1/.4 = 2.5$ weeks between successive errors. Letting x denote the number of weeks elapsing between successive errors, the equation of the exponential distribution describing x is $f(x) = \lambda e^{-\lambda x} = .4e^{-.4x}$. For example, the probability that the time between successive errors will be between 1 and 2 weeks is

$$\begin{aligned} P(1 \le x \le 2) &= e^{-\lambda a} - e^{-\lambda b} = e^{-\lambda(1)} - e^{-\lambda(2)} \\ &= e^{-.4(1)} - e^{-.4(2)} = e^{-.4} - e^{-.8} \\ &= .6703 - .4493 = .221 \end{aligned}$$

EXAMPLE 6.11 Emergency Room Arrivals

Suppose that the number of people who arrive at a hospital emergency room during a given time period has a Poisson distribution. It follows that the time, x, between successive arrivals of people to the emergency room has an exponential distribution. Furthermore, historical records indicate that the mean time between successive arrivals of people to the emergency room is seven minutes. Therefore, $\mu_x = 1/\lambda = 7$, which implies that $\lambda = 1/7 = .14286$. Noting that $\sigma_x = 1/\lambda = 7$, it follows that

$$\mu_x - \sigma_x = 7 - 7 = 0 \qquad \text{and} \qquad \mu_x + \sigma_x = 7 + 7 = 14$$

Therefore, the probability that the time between successive arrivals of people to the emergency room will be within (plus or minus) one standard deviation of the mean interarrival time is

$$\begin{aligned} P(0 \le x \le 14) &= e^{-\lambda a} - e^{-\lambda b} \\ &= e^{-(.14286)(0)} - e^{-(.14286)(14)} \\ &= 1 - .1353 \\ &= .8647 \end{aligned}$$

To conclude this section we note that the exponential and related Poisson distributions are useful in analyzing waiting lines, or **queues.** In general, **queueing theory** attempts to determine the number of servers (for example, doctors in an emergency room) that strikes an optimal balance between the time customers wait for service and the cost of providing service. The reader is referred to any textbook on management science or operations research for a discussion of queueing theory.

Exercises for Section 6.5

CONCEPTS

connect

6.51 Give two examples of situations in which the exponential distribution might be used appropriately. In each case, define the random variable having an exponential distribution.

6.52 State the formula for the exponential probability curve. Define each symbol in the formula.

6.53 Explain the relationship between the Poisson and exponential distributions.

METHODS AND APPLICATIONS

6.54 Suppose that the random variable x has an exponential distribution with $\lambda = 2$.

a Write the formula for the exponential probability curve of x. What are the possible values of x?

b Sketch the probability curve.

c Find $P(x \leq 1)$.

d Find $P(.25 \leq x \leq 1)$.

e Find $P(x \geq 2)$.

f Calculate the mean, μ_x, the variance, σ_x^2, and the standard deviation, σ_x, of the exponential distribution of x.

g Find the probability that x will be in the interval $[\mu_x \pm 2\sigma_x]$.

6.55 Repeat Exercise 6.54 with $\lambda = 3$.

6.56 Recall in Exercise 5.32 (page 212) that the number of customer arrivals at a bank's drive-up window in a 15-minute period is Poisson distributed with a mean of seven customer arrivals per 15-minute period. Define the random variable x to be the time (in minutes) between successive customer arrivals at the bank's drive-up window.

a Write the formula for the exponential probability curve of x.

b Sketch the probability curve of x.

c Find the probability that the time between arrivals is:

(1) Between one and two minutes.

(2) Less than one minute.

(3) More than three minutes.

(4) Between 1/2 and $3^1/_2$ minutes.

d Calculate μ_x, σ_x^2, and σ_x.

e Find the probability that the time between arrivals falls within one standard deviation of the mean; within two standard deviations of the mean.

6.57 The length of a particular telemarketing phone call, x, has an exponential distribution with mean equal to 1.5 minutes.

a Write the formula for the exponential probability curve of x.

b Sketch the probability curve of x.

c Find the probability that the length of a randomly selected call will be:

(1) No more than three minutes.

(2) Between one and two minutes.

(3) More than four minutes.

(4) Less than 30 seconds.

6.58 The maintenance department in a factory claims that the number of breakdowns of a particular machine follows a Poisson distribution with a mean of two breakdowns every 500 hours. Let x denote the time (in hours) between successive breakdowns.

a Find λ and μ_x.

b Write the formula for the exponential probability curve of x.

c Sketch the probability curve.

d Assuming that the maintenance department's claim is true, find the probability that the time between successive breakdowns is at most five hours.

e Assuming that the maintenance department's claim is true, find the probability that the time between successive breakdowns is between 100 and 300 hours.

f Suppose that the machine breaks down five hours after its most recent breakdown. Based on your answer to part *d*, do you believe the maintenance department's claim? Explain.

6.59 Suppose that the number of accidents occurring in an industrial plant is described by a Poisson distribution with an average of one accident per month. Let x denote the time (in months) between successive accidents.

a Find the probability that the time between successive accidents is:

(1) More than two months.

(2) Between one and two months.

(3) Less than one week (1/4 of a month).

b Suppose that an accident occurs less than one week after the plant's most recent accident. Would you consider this event unusual enough to warrant special investigation? Explain.

6.6 The Normal Probability Plot (Optional)

LO6-8 Use a normal probability plot to help decide whether data come from a normal distribution (Optional).

The **normal probability plot** is a graphic that is used to visually check whether sample data come from a normal distribution. In order to illustrate the construction and interpretation of a normal probability plot, consider the e-billing case and suppose that the trucking company operates in three regions of the country—the north, central, and south regions. In each region, 24 invoices are randomly selected and the payment time for each sampled invoice is found. The payment times obtained in each region are given in Table 6.3, along with MINITAB side-by-side box plots of the data. Examination of the data and box plots indicates that the payment times for the central region are skewed to the left, while the payment times for the south region are skewed to the right. The box plot of the payment times for the north region, along with the dot plot of these payment times in Figure 6.28, indicate that the payment times for the north region are approximately normally distributed.

We will begin by constructing a normal probability plot for the payment times from the north region. We first arrange the payment times in order from smallest to largest. The ordered payment times are shown in column (1) of Table 6.4 on the next page. Next, for each ordered payment time, we compute the quantity $i/(n + 1)$, where i denotes the observation's position in the ordered list of data and n denotes the sample size. For instance, for the first and second ordered payment times, we compute $1/(24 + 1) = 1/25 = .04$ and $2/(24 + 1) = 2/25 = .08$. Similarly, for the last (24th) ordered payment time, we compute $24/(24 + 1) = 24/25 = .96$. The positions ($i$ values) of all 24 payment times are given in column (2) of Table 6.4, and the corresponding values of $i/(n + 1)$ are given in column (3) of this table. We continue by computing what is called the **standardized normal quantile value** for each ordered payment time. This value (denoted O_i) is the z value that

TABLE 6.3 Twenty-four Randomly Selected Payment Times for Each of Three Geographical Regions in the United States DS RegPayTime

North Region	Central Region	South Region
26	26	28
27	28	31
21	21	21
22	22	23
22	23	23
23	24	24
27	27	29
20	19	20
22	22	22
29	29	36
18	15	19
24	25	25
28	28	33
26	26	27
21	20	21
32	29	44
23	24	24
24	25	26
25	25	27
15	7	19
17	12	19
19	18	20
34	29	50
30	29	39

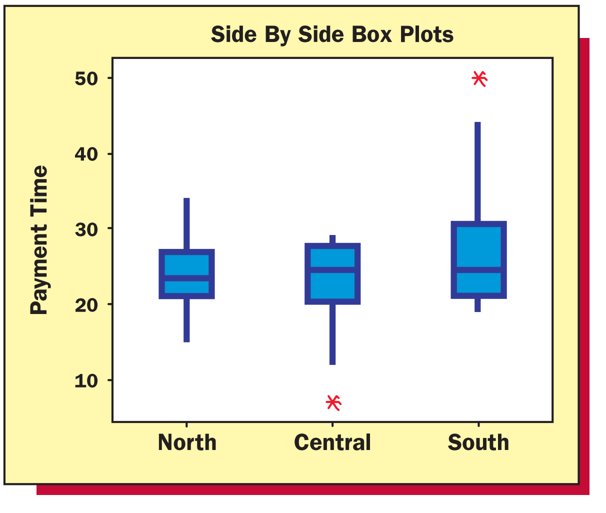


FIGURE 6.28 Dot Plot of the Payment Times for the North Region

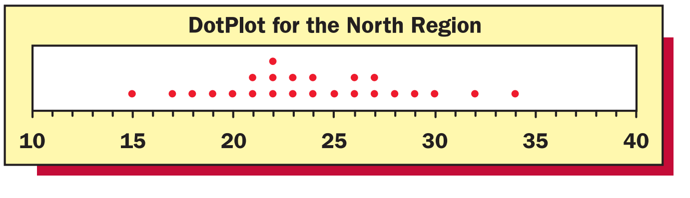
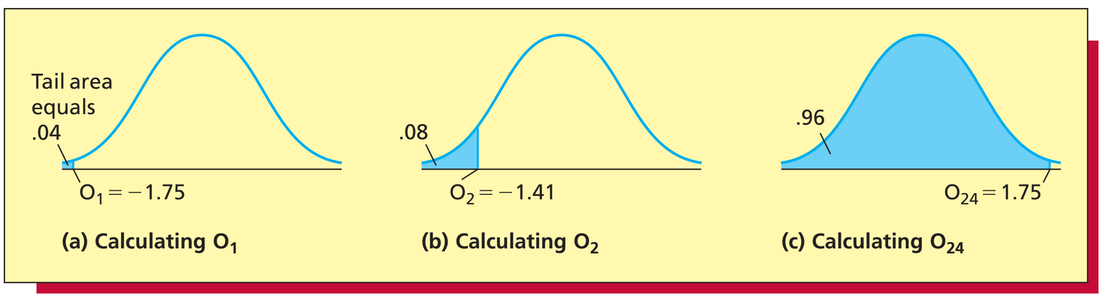


TABLE 6.4 Calculations for Normal Probability Plots in the e-billing Example

Ordered North Region Payment Times Column (1)	Observation Number (i) Column (2)	Area $i/(n+1)$ Column (3)	*z* value O_i Column (4)	Ordered Central Region Payment Times Column (5)	Ordered South Region Payment Times Column (6)
15	1	0.04	−1.75	7	19
17	2	0.08	−1.41	12	19
18	3	0.12	−1.18	15	19
19	4	0.16	−0.99	18	20
20	5	0.2	−0.84	19	20
21	6	0.24	−0.71	20	21
21	7	0.28	−0.58	21	21
22	8	0.32	−0.47	22	22
22	9	0.36	−0.36	22	23
22	10	0.4	−0.25	23	23
23	11	0.44	−0.15	24	24
23	12	0.48	−0.05	24	24
24	13	0.52	0.05	25	25
24	14	0.56	0.15	25	26
25	15	0.6	0.25	25	27
26	16	0.64	0.36	26	27
26	17	0.68	0.47	26	28
27	18	0.72	0.58	27	29
27	19	0.76	0.71	28	31
28	20	0.8	0.84	28	33
29	21	0.84	0.99	29	36
30	22	0.88	1.18	29	39
32	23	0.92	1.41	29	44
34	24	0.96	1.75	29	50

FIGURE 6.29 Calculating Standardized Normal Quantile Values



gives an area of $i/(n + 1)$ to its left under the standard normal curve. Figure 6.29 illustrates finding O_1, O_2, and O_{24}. For instance, O_1—the standardized normal quantile value corresponding to the first ordered residual—is the z value that gives an area of $1/(24 + 1) = .04$ to its left under the standard normal curve. As shown in Figure 6.29(a), the z value (to two decimal places) that gives a left-hand tail area closest to .04 is $O_1 = -1.75$. Similarly, O_2 is the z value that gives an area of $2/(24 + 1) = .08$ to its left under the standard normal curve. As shown in Figure 6.29(b), the z value (to two decimal places) that gives a left-hand tail area closest to .08 is $O_2 = -1.41$. As a final example, Figure 6.29(c) shows that O_{24}, the z value that gives an area of $24/(24 + 1) = .96$ to its left under the standard normal curve, is 1.75. The standardized normal quantile values corresponding to the 24 ordered payment times are given in column (4) of Table 6.4. Finally, we obtain the **normal probability plot** by plotting the 24 ordered payment times on the vertical axis versus the corresponding standardized normal quantile values (O_i values) on the horizontal axis. Figure 6.30 gives an Excel add-in (MegaStat) output of this normal probability plot.

In order to interpret the normal plot, notice that, although the areas in column (3) of Table 6.4 (that is, the $i/(n + 1)$ values: .04, .08, .12, etc.) are equally spaced, the z values corresponding to

FIGURE 6.30 Excel Add-in (MegaStat) Normal Probability Plot for the North Region: Approximate Normality

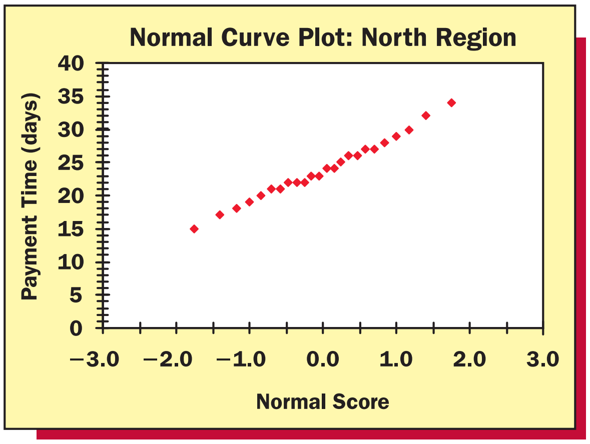


FIGURE 6.31 Excel Add-in (MegaStat) Normal Probability Plot for the Central Region: Data Skewed to the Left

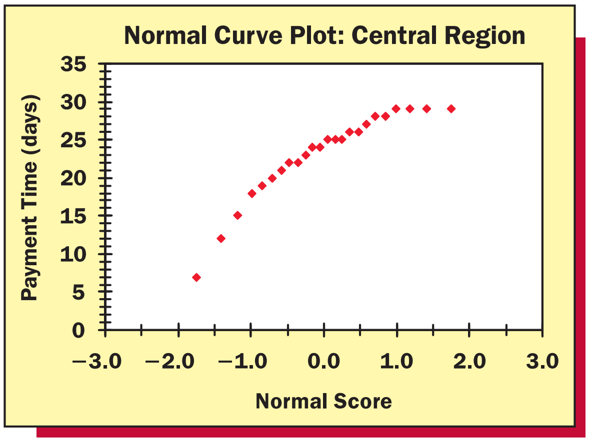
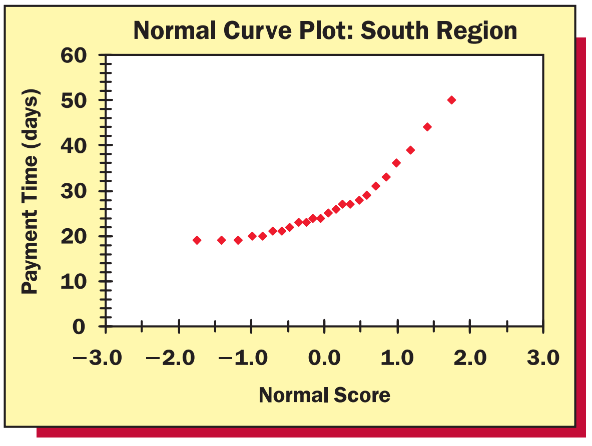


FIGURE 6.32 Excel Add-in (MegaStat) Normal Probability Plot for the South Region: Data Skewed to the Right



these areas are not equally spaced. Because of the mound-shaped nature of the standard normal curve, the negative z values get closer together as they get closer to the mean ($z = 0$) and the positive z values get farther apart as they get farther from the mean (more positive). If the distances between the payment times behave the same way as the distances between the z values—that is, if the distances between the payment times are proportional to the distances between the z values—then the normal probability plot will be a straight line. This would suggest that the payment times are normally distributed. Examining Figure 6.30, the normal probability plot for the payment times from the north region is approximately a straight line and, therefore, it is reasonable to assume that these payment times are approximately normally distributed.

Column (5) of Table 6.4 gives the ordered payment times for the central region, and Figure 6.31 plots these values versus the standardized normal quantile values in column (4). The resulting normal probability plot for the central region has a nonlinear appearance. The plot points rise more steeply at first and then continue to increase at a decreasing rate. This pattern indicates that the payment times for the central region are skewed to the left. Here the rapidly rising points at the beginning of the plot are due to the payment times being farther apart in the left tail of the distribution. Column (6) of Table 6.4 gives the ordered payment times for the south region, and Figure 6.32 gives the normal probability plot for this region. This plot also has a nonlinear appearance. The points rise slowly at first and then increase at an increasing rate. This pattern indicates that the payment times for the south region are skewed to the right. Here the rapidly rising points on the right side of the plot are due to the payment times being farther apart in the right tail of the distribution.

In the following box, we summarize how to construct and interpret a normal probability plot.

Normal Probability Plots

1. Order the values in the data from smallest to largest.
2. For each observation compute the area $i/(n + 1)$, where i denotes the position of the observation in the ordered listing and n is the number of observations.
3. Compute the standardized normal quantile value O_i for each observation. This is the z value that gives an area of $i/(n + 1)$ to its left under the standard normal curve.
4. Plot the ordered data values versus the standardized normal quantile values.
5. If the resulting normal probability plot has a straight line appearance, it is reasonable to assume that the data come from a normal distribution.

Exercises for Section 6.6

CONCEPTS

connect

6.60 Discuss how a normal probability plot is constructed.

6.61 If a normal probability plot has the appearance of a straight line, what should we conclude?

METHODS AND APPLICATIONS

6.62 Consider the sample of 12 incomes given in Example 3.2 (page 105).
- **a** Sort the income data from smallest to largest, and compute $i/(n + 1)$ for each observation.
- **b** Compute the standardized normal quantile value O_i for each observation.
- **c** Graph the normal probability plot for the salary data and interpret this plot. Does the plot indicate that the data are skewed? Explain. DS Incomes

6.63 Consider the 20 DVD satisfaction ratings given on page 123. Construct a normal probability plot for these data and interpret the plot. DS DVDSat

6.64 A normal probability plot can be constructed using MINITAB. Use the selections Stat : Basic Statistics : Normality test, and select the data to be analyzed. Although the MINITAB plot is slightly different from the plot outlined in this section, its interpretation is the same. Use MINITAB to construct a normal probability plot of the gas mileage data in Table 3.1 (page 103). Interpret the plot. DS GasMiles

Chapter Summary

In this chapter we have discussed **continuous probability distributions.** We began by learning that **a continuous probability distribution is described by a continuous probability curve** and that in this context **probabilities are areas under the probability curve.** We next studied two important continuous probability distributions—**the uniform distribution** and **the normal distribution.** In particular, we concentrated on the normal distribution, which is the most important continuous probability distribution. We learned about the properties of the normal curve, and we saw how to use a **normal table** to find various areas under a normal curve. We then demonstrated how we can use a normal curve probability to make a statistical inference. We continued with an optional section that explained how we can use a normal curve probability to approximate a binomial probability. Then we presented an optional section that discussed another important continuous probability distribution—**the exponential distribution,** and we saw how this distribution is related to the Poisson distribution. Finally, we concluded this chapter with an optional section that explained how to use a **normal probability plot** to decide whether data come from a normal distribution.

Glossary of Terms

continuous probability distribution (or **probability curve**): A curve that is defined so that the probability that a random variable will be in a specified interval of numbers is the area under the curve corresponding to the interval. (pages 225, 226)

cumulative normal table: A table in which we can look up areas under the standard normal curve. (pages 232–234)

exponential distribution: A probability distribution that describes the time or space between successive occurrences of an event when the number of times the event occurs over an interval of time or space is described by a Poisson distribution. (page 252)

normal probability distribution: The most important continuous probability distribution. Its probability curve is the *bell-shaped* normal curve. (page 230)

normal probability plot: A graphic used to visually check whether sample data come from a normal distribution. (page 255)

queueing theory: A methodology that attempts to determine the number of servers that strikes an optimal balance between the time customers wait for service and the cost of providing service. (page 253)

standard normal distribution (or curve): A normal distribution (or curve) having mean 0 and standard deviation 1. (page 232)

uniform distribution: A continuous probability distribution having a rectangular shape that says the probability is distributed evenly (or uniformly) over an interval of numbers. (page 228)

z_α point: The point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to α. (page 241)

$-z_\alpha$ point: The point on the horizontal axis under the standard normal curve that gives a left-hand tail area equal to α. (page 242)

z value: A value that tells us the number of standard deviations that a value x is from the mean of a normal curve. If the z value is positive, then x is above the mean. If the z value is negative, then x is below the mean. (page 232)

Important Formulas

The uniform probability curve: page 228

Mean and standard deviation of a uniform distribution: page 228

The normal probability curve: page 230

z values: page 232

Finding normal probabilities: page 238

Normal approximation to the binomial distribution: page 248

The exponential probability curve: page 252

Mean and standard deviation of an exponential distribution: page 252

Constructing a normal probability plot: page 258

Supplementary Exercises

6.65 In a bottle-filling process, the amount of drink injected into 16 oz bottles is normally distributed with a mean of 16 oz and a standard deviation of .02 oz. Bottles containing less than 15.95 oz do not meet the bottler's quality standard. What percentage of filled bottles do not meet the standard?

6.66 In a murder trial in Los Angeles, a shoe expert stated that the range of heights of men with a size 12 shoe is 71 inches to 76 inches. Suppose the heights of all men wearing size 12 shoes are normally distributed with a mean of 73.5 inches and a standard deviation of 1 inch. What is the probability that a randomly selected man who wears a size 12 shoe:

- **a** Has a height outside the range 71 inches to 76 inches?
- **b** Is 74 inches or taller?
- **c** Is shorter than 70.5 inches?

6.67 In the movie *Forrest Gump,* the public school required an IQ of at least 80 for admittance.

- **a** If IQ test scores are normally distributed with mean 100 and standard deviation 16, what percentage of people would qualify for admittance to the school?
- **b** If the public school wishes 95 percent of all children to qualify for admittance, what minimum IQ test score should be required for admittance?

6.68 The amount of sales tax paid on a purchase is rounded to the nearest cent. Assume that the round-off error is uniformly distributed in the interval $-.5$ to .5 cent.

- **a** Write the formula for the probability curve describing the round-off error.
- **b** Graph the probability curve describing the round-off error.
- **c** What is the probability that the round-off error exceeds .3 cent or is less than $-.3$ cent?
- **d** What is the probability that the round-off error exceeds .1 cent or is less than $-.1$ cent?
- **e** Find the mean and the standard deviation of the round-off error.
- **f** Find the probability that the round-off error will be within one standard deviation of the mean.

6.69 A *consensus forecast* is the average of a large number of individual analysts' forecasts. Suppose the individual forecasts for a particular interest rate are normally distributed with a mean of 5.0 percent and a standard deviation of 1.2 percent. A single analyst is randomly selected. Find the probability that his/her forecast is:

- **a** At least 3.5 percent.
- **b** At most 6 percent.
- **c** Between 3.5 percent and 6 percent.

6.70 Recall from Exercise 6.69 that individual forecasts of a particular interest rate are normally distributed with a mean of 5 percent and a standard deviation of 1.2 percent.

a What percentage of individual forecasts are at or below the 10th percentile of the distribution of forecasts? What percentage are at or above the 10th percentile? Find the 10th percentile of the distribution of individual forecasts.

b Find the first quartile, Q_1, and the third quartile, Q_3, of the distribution of individual forecasts.

6.71 The scores on the entrance exam at a well-known, exclusive law school are normally distributed with a mean score of 200 and a standard deviation equal to 50. At what value should the lowest passing score be set if the school wishes only 2.5 percent of those taking the test to pass?

6.72 A machine is used to cut a metal automobile part to its desired length. The machine can be set so that the mean length of the part will be any value that is desired. The standard deviation of the lengths always runs at .02 inch. Where should the mean be set if we want only .4 percent of the parts cut by the machine to be shorter than 15 inches long?

6.73 A motel accepts 325 reservations for 300 rooms on July 1, expecting 10 percent no-shows on average from past records. Use the normal approximation to the binomial to find the probability that all guests who arrive on July 1 will receive a room.

6.74 Suppose a software company finds that the number of errors in its software per 1,000 lines of code is described by a Poisson distribution. Furthermore, it is found that there is an average of four errors per 1,000 lines of code. Letting x denote the number of lines of code between successive errors:

a Find the probability that there will be at least 400 lines of code between successive errors in the company's software.

b Find the probability that there will be no more than 100 lines of code between successive errors in the company's software.

6.75 THE INVESTMENT CASE DS InvestRet

For each investment class in Table 3.9 (page 141), assume that future returns are normally distributed with the population mean and standard deviation given in Table 3.9. Based on this assumption:

a For each investment class, find the probability of a return that is less than zero (that is, find the probability of a loss). Is your answer reasonable for all investment classes? Explain.

b For each investment class, find the probability of a return that is:

(1) Greater than 5 percent.
(2) Greater than 10 percent.
(3) Greater than 20 percent.
(4) Greater than 50 percent.

c For which investment classes is the probability of a return greater than 50 percent essentially zero? For which investment classes is the probability of such a return greater than 1 percent? Greater than 5 percent?

d For which investment classes is the probability of a loss essentially zero? For which investment classes is the probability of a loss greater than 1 percent? Greater than 10 percent? Greater than 20 percent?

6.76 The daily water consumption for an Ohio community is normally distributed with a mean consumption of 800,000 gallons and a standard deviation of 80,000 gallons. The community water system will experience a noticeable drop in water pressure when the daily water consumption exceeds 984,000 gallons. What is the probability of experiencing such a drop in water pressure?

6.77 Suppose the times required for a cable company to fix cable problems in its customers' homes are uniformly distributed between 10 minutes and 25 minutes. What is the probability that a randomly selected cable repair visit will take at least 15 minutes?

6.78 Suppose the waiting time to get food after placing an order at a fast-food restaurant is exponentially distributed with a mean of 60 seconds. If a randomly selected customer orders food at the restaurant, what is the probability that the customer will wait at least:

a 90 seconds?

b Two minutes?

6.79 Net interest margin—often referred to as *spread*—is the difference between the rate banks pay on deposits and the rate they charge for loans. Suppose that the net interest margins for all U.S. banks are normally distributed with a mean of 4.15 percent and a standard deviation of .5 percent.

a Find the probability that a randomly selected U.S. bank will have a net interest margin that exceeds 5.40 percent.

b Find the probability that a randomly selected U.S. bank will have a net interest margin less than 4.40 percent.

c A bank wants its net interest margin to be less than the net interest margins of 95 percent of all U.S. banks. Where should the bank's net interest margin be set?

6.80 In an article in *Advertising Age,* Nancy Giges studies global spending patterns. Giges presents data concerning the percentage of adults in various countries who have purchased various consumer items (such as soft drinks, athletic footware, blue jeans, beer, and so on) in the past three months.

a Suppose we wish to justify the claim that fewer than 50 percent of adults in Germany have purchased blue jeans in the past three months. The survey reported by Giges found that 45 percent of the respondents in Germany had purchased blue jeans in the past three months.

Assume that a random sample of 400 German adults was employed, and let p be the proportion of all German adults who have purchased blue jeans in the past three months. If, for the sake of argument, we assume that $p = .5$, use the normal approximation to the binomial distribution to calculate the probability that 45 percent or fewer of 400 randomly selected German adults would have purchased blue jeans in the past three months. Note: Because 45 percent of 400 is 180, you should calculate the probability that 180 or fewer of 400 randomly selected German adults would have purchased blue jeans in the past three months.

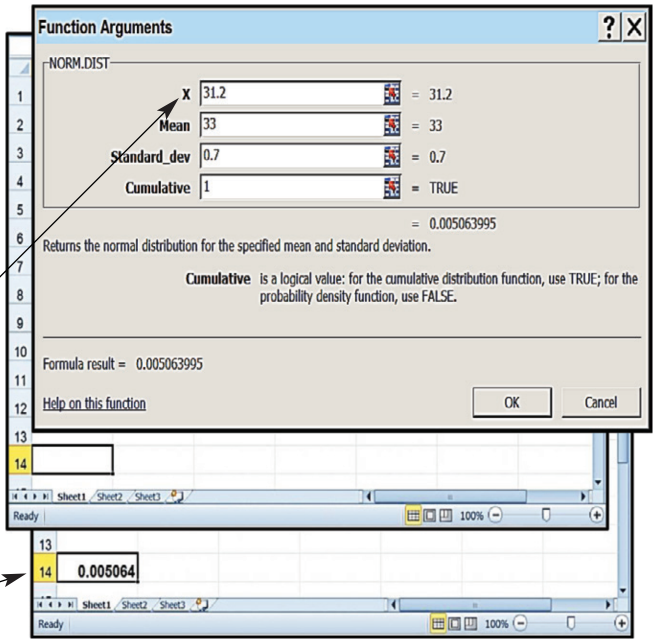
b Based on the probability you computed in part *a*, would you conclude that p is really less than .5? That is, would you conclude that fewer than 50 percent of adults in Germany have purchased blue jeans in the past three months? Explain.

6.81 Assume that the ages for first marriages are normally distributed with a mean of 26 years and a standard deviation of 4 years. What is the probability that a person getting married for the first time is in his or her twenties?

Appendix 6.1 ■ Normal Distribution Using Excel

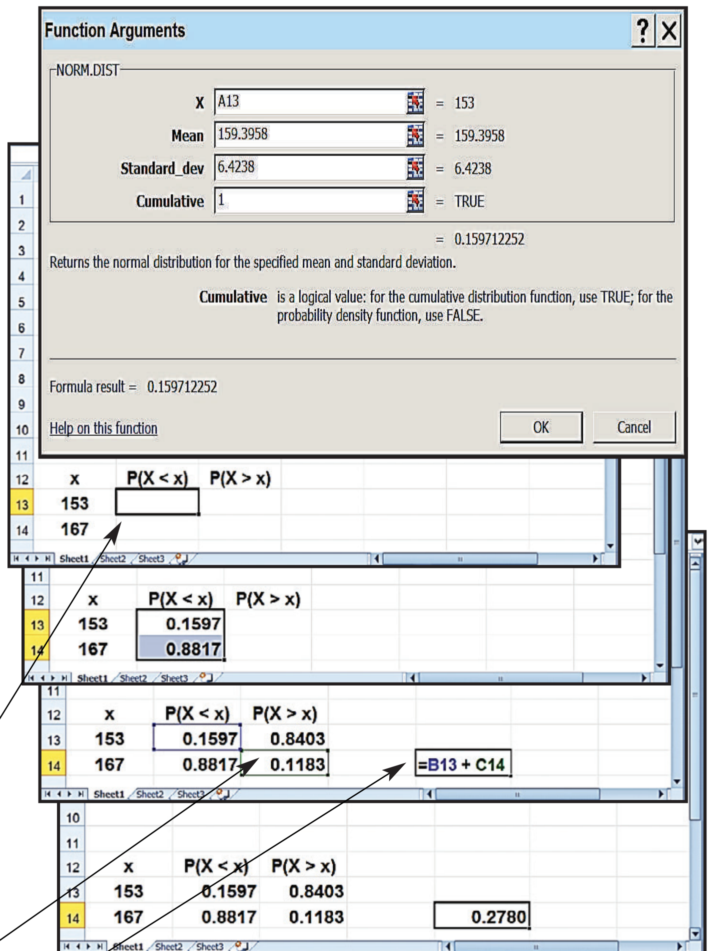
Normal probability $P(X < 31.2)$ in Example 6.3 (page 239):

- Click in the cell where you wish to place the answer. Here we have clicked in cell A14. Then select the Insert Function button $\boxed{f_x}$ from the Excel toolbar.
- In the Insert Function dialog box, select Statistical from the "Or select a category:" menu, select NORM.DIST from the "Select a function:" menu, and click OK.
- In the NORM.DIST Function Arguments dialog box, enter the value 31.2 in the X window.
- Enter the value 33 in the Mean window.
- Enter the value 0.7 in the Standard_dev window.
- Enter the value 1 in the Cumulative window.
- Click OK in the NORM.DIST Function Arguments dialog box.
- When you click OK in this dialog box, the answer will be placed in cell A14.



Normal probability $P(X < 153 \text{ or } X > 167)$ in Example 6.4 (page 240):

- Enter the headings—x, P(X < x), P(X > x)–in the spreadsheet where you wish the results to be placed. Here we will enter these headings in cells A12, B12, and C12. The calculated results will be placed below the headings.
- In cells A13 and A14, enter the values 153 and 167.
- Click in cell B13 and select the Insert Function button $\boxed{f_x}$ from the Excel toolbar.
- In the Insert Function dialog box, select Statistical from the "Or select a category:" menu, select NORM.DIST from the "Select a function:" menu, and click OK.
- In the NORM.DIST Function Arguments dialog box, enter the cell location A13 in the X window.
- Enter the value 159.3958 in the Mean window.
- Enter the value 6.4238 in the Standard_dev window.
- Enter the value 1 in the Cumulative window.
- Click OK in the NORM.DIST Function Arguments dialog box.
- When you click OK, the result for $P(X < 153)$ will be placed in cell B13. Double-click the drag-handle (in the lower right corner) of cell B13 to automatically extend the cell formula of B13 through cell B14.
- In cells C13 and C14, enter the formulas $=1-\text{B13}$ and $=1-\text{B14}$. The results for $P(X > 153)$ and $P(X > 167)$ will be placed in cells C13 and C14.
- In cell E14, enter the formula = B13 + C14.

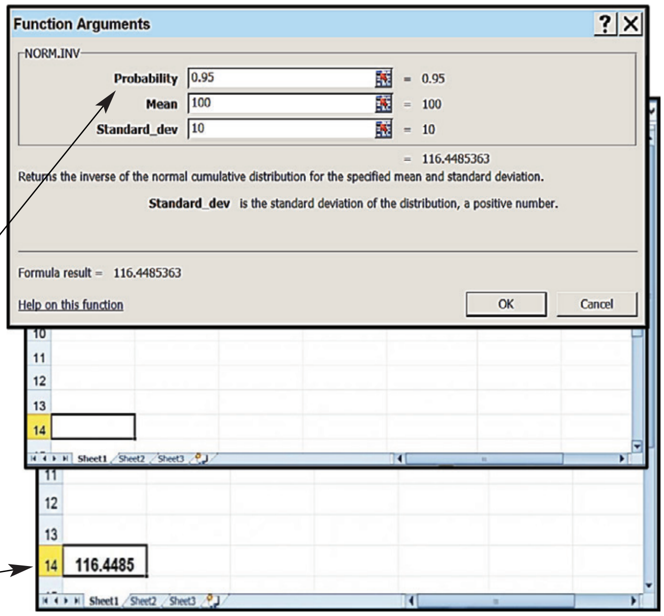


The desired probability is in cell E14, the sum of the lower tail probability for 153 and the upper tail probability for 167. This value differs slightly from the value in Example 6.4 because Excel carries out probability calculations to higher precision than can be achieved using normal probability tables.

Inverse normal probability *st* such that $P(X > st) = 0.05$ in Example 6.5 (pages 241–242):

- Click in the cell where you wish the answer to be placed. Here we will click in cell A14. Select the Insert Function button $\boxed{f_x}$ from the Excel toolbar.
- In the Insert Function dialog box, select Statistical from the "Or select a category:" menu, select NORM.INV from the "Select a function:" menu, and click OK.
- In the NORM.INV Function Arguments dialog box, enter the value 0.95 in the Probability window; that is,

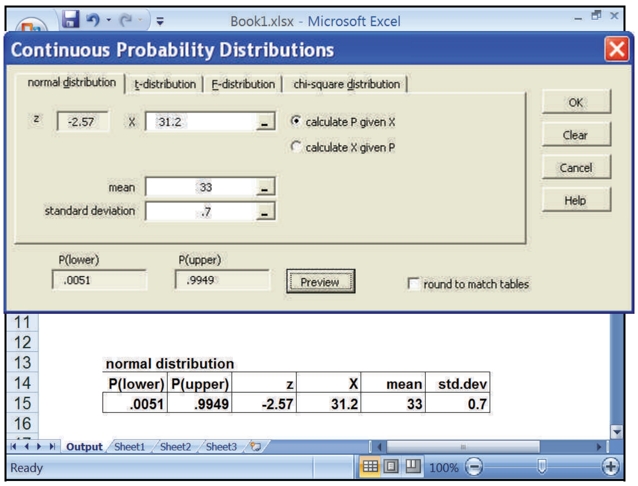
  $[P(X < st) = 0.95$ when $P(X > st) = 0.05.]$
- Enter the value 100 in the Mean window.
- Enter the value 10 in the Standard_dev window.
- Click OK in the NORM.INV Function Arguments dialog box.
- When you click OK, the answer is placed in cell A14.



Appendix 6.2 ■ Normal Distribution Using MegaStat

Normal probability $P(X < 31.2)$ in Example 6.3 (page 239):

- Select **Add-ins : MegaStat : Probability : Continuous Probability Distributions**
- In the "Continuous Probability Distributions" dialog box, select the normal distribution tab.
- Enter the distribution mean (here equal to 33) and the distribution standard deviation (here equal to 0.7) in the appropriate boxes.
- Enter the value of *x* (here equal to 31.2) into the "Calculate p given x" window.
- Click OK in the "Continuous Probability Distributions" dialog box.
- The output includes **P(lower),** which is the area under the specified normal curve below the given value of *x*, and **P(upper),** which is the area under the specified normal curve above the given value of *x*. The value of *z* corresponding to the specified value of *x* is also included. In this case, $P(X < 31.2)$ equals $P(\text{lower}) = .0051$.
- (Optional) Click on the preview button to see the values of P(lower) and P(upper) before obtaining results in the Output worksheet.

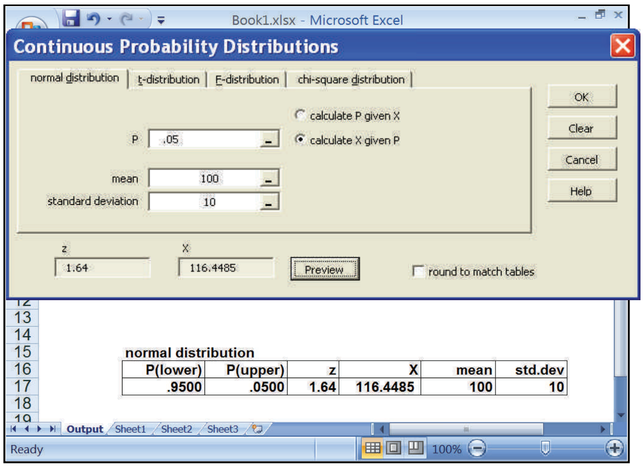


Note that if a **standard normal distribution** is specified, 0 is entered in the mean box and 1 is entered in the standard deviation box—the "calculate P given X" box will read "Calculate P given z." In this case, when we enter a value of z in the "Calculate P given z" box, P(lower) and P(upper) are, respectively, the areas below and above the specified value of z under the standard normal curve.

Normal probability $P(X < 153$ or $X > 167)$ in Example 6.4 on page 240. Enter 159.3958 into the Mean box and enter 6.4238 into the Standard Deviation box. Find P(lower) corresponding to 153 and find P(upper) corresponding to 167. When these values are placed in the output worksheet, use a simple Excel cell formula to add them together.

Inverse normal probability *st* such that $P(X > st) = 0.05$ in Example 6.5 on pages 241–242:

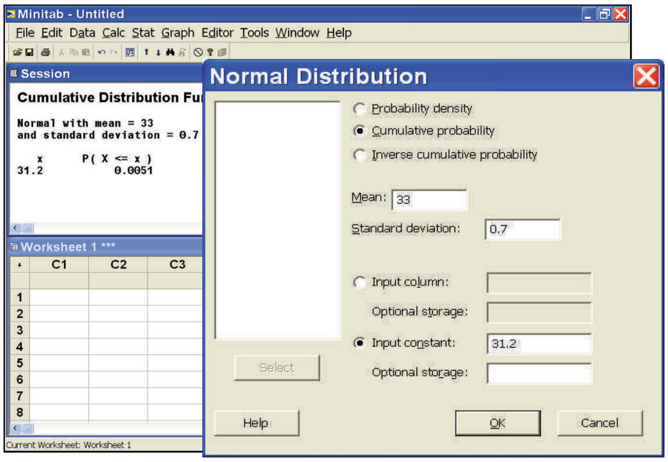
- Select **Add-ins : MegaStat : Probability : Continuous Probability Distributions**
- Enter 100 into the Mean box and enter 10 into the Standard deviation box.
- Select the "Calculate x given P" option.
- Enter 0.05 into the P box. This is the area under the normal curve we want to have above *st* (that is, above the desired value of x).
- Click OK in the "Continuous Probability Distributions" dialog box.
- The output includes P(lower) and P(upper)—as defined above—as well as the desired value of x (in this case x equals 116.4485).



Appendix 6.3 ■ Normal Distribution Using MINITAB

Normal probability $P(X \leq 31.2)$ in Example 6.3 (page 239):

- Select **Calc : Probability Distributions : Normal.**
- In the Normal Distribution dialog box, select the Cumulative probability option.
- In the Mean window, enter 33.
- In the Standard deviation window, enter 0.7.
- Click on the "Input constant" option and enter 31.2 in the "Input constant" window.
- Click OK in Normal Distribution dialog box to see the desired probability in the Session window.

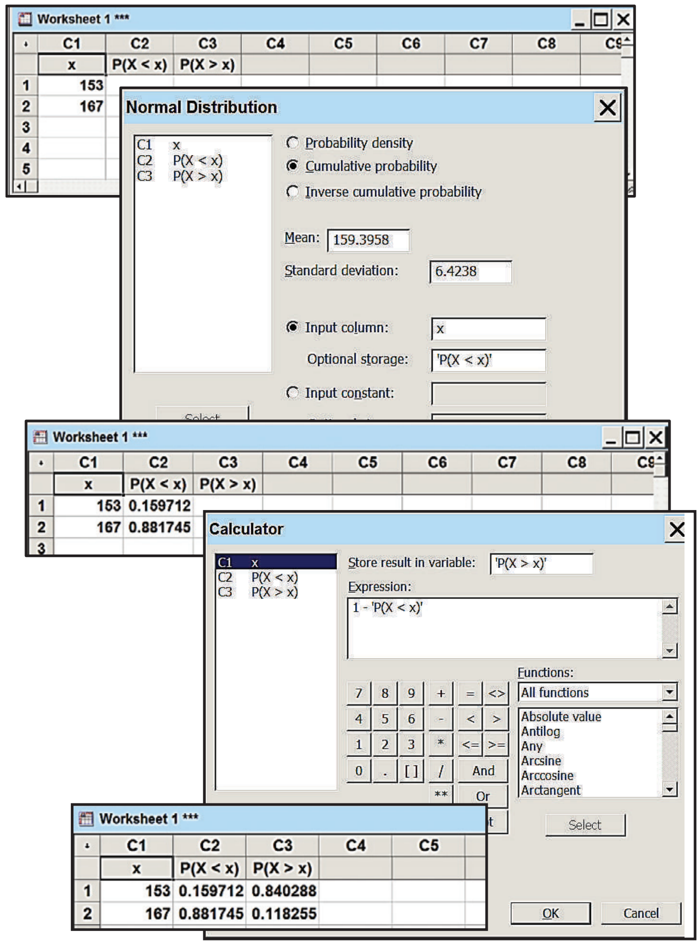


Normal probability $P(X < 153$ or $X > 167)$ in Example 6.4 (page 240):

- In columns C1, C2, and C3, enter the variable names—x, P(X < x), and P(X > x).
- In column C1, enter the values 153 and 167.
- Select **Calc : Probability Distributions : Normal.**
- In the Normal Distribution dialog box, select the Cumulative probability option.
- In the Mean window, enter 159.3958.
- In the Standard deviation window, enter 6.4238.
- Click the "Input column" option, enter x in the "Input column" window, and enter 'P(X < x)' in the "Optional storage" window.
- Click OK in Normal Distribution dialog box.

- Select **Calc : Calculator.**
- In the Calculator dialog box, enter 'P(X > x)' in the "Store result in variable" window.
- Enter 1 − 'P(X < x)' in the Expression window.
- Click OK in the Calculator dialog box.

The desired probability is the sum of the lower tail probability for 153 and the upper tail probability for 167 or 0.159712 + 0.118255 = 0.277967. This value differs slightly from the value in Example 6.4 because MINITAB carries out probability calculations to higher precision than can be achieved using normal probability tables.



Inverse normal probability to find the number of units stocked, *st*, such that $P(X > st) = 0.05$ in Example 6.5 (pages 241–242):

- Select **Calc : Probability Distributions : Normal.**
- In the Normal Distribution dialog box, select the Inverse cumulative probability option.
- In the Mean window, enter 100.
- In the Standard deviation window, enter 10.
- Click the "Input constant" option and enter 0.95 in the "Input constant" window. That is,

  $P(X \leq st) = 0.95$ when $P(X > st) = 0.05$.
- Click OK in the Normal Distribution dialog box to see the desired value of *st* in the Session window.