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Hypothesis Testing

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Chapter 8

Hypothesis Testing

Hypothesis Testing

8.1 Null and Alternative Hypotheses and Errors in Testing

8.2 z Tests about a Population Mean (σ Known): One-Sided Alternatives

8.3 z Tests about a Population Mean (σ Known): Two-Sided Alternatives

8.4 t Tests about a Population Mean (σ Unknown)

Hypothesis Testing Continued

Null and Alternative Hypotheses

- The *null hypothesis*, denoted H_0, is a statement of the basic proposition being tested.
  - The statement generally represents the *status quo* and is not rejected unless there is convincing sample evidence that it is false.
- The *alternative* or *research hypothesis*, denoted H_a, is an alternative (to the null hypothesis) statement that will be accepted only if there is convincing sample evidence that it is true

Types of Hypotheses

- One-Sided, "Greater Than" Alternative

  $H_0: \mu \leq \mu_0$ vs. $H_a: \mu > \mu_0$

- One-Sided, "Less Than" Alternative

  $H_0 : \mu \geq \mu_0$ vs. $H_a : \mu < \mu_0$

- Two-Sided, "Not Equal To" Alternative

  $H_0 : \mu = \mu_0$ vs. $H_a : \mu \neq \mu_0$

where μ_0 is a given constant value (with the appropriate units) that is a comparative value

Types of Decisions

- As a result of testing H_0 vs. H_a, will have to decide either of the following decisions for the null hypothesis H_0:

- Do not reject H_0
  - A weaker statement than "accepting H_0"
  - But you are rejecting the alternative H_a

OR

- Reject H_0

Examples of Hypothesis

- Example 8.1. The trash bag case
  - H_0: $\mu \leq 50$
  - versus H_a: $\mu > 50$
- Example 8.2. The accounts receivable case
  - H_0: $\mu \geq 19.5$
  - versus H_a: $\mu < 19.5$
- Example 8.3. The camshaft case
  - H_0: $\mu = 4.5$
  - versus H_a: $\mu \neq 19.5$

Test Statistic

- In order to test H_0 vs. H_a, use the "test statistic"

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

where μ_0 is the given value (often the "claimed to be true") and ☒ is the mean of a sample

- *z* measures the distance between μ_0 and ☒ on the sampling distribution of the sample mean
- If the population is normal or the sample size is large*, then the test statistic *z* follows a normal distribution

* $n \geq 30$, by the Central Limit Theorem

Type I and Type II Errors

Type I Error: Rejecting H_0 when it is true

Type II Error: Failing to reject H_0 when it is false

State of Nature

Conclusion	H_0 True	H_0 False
Reject H_0	**Type I Error**	Correct Decision
Do not Reject H_0	Correct Decision	**Type II Error**

Error Probabilities

Type I Error: Rejecting H_0 when it is true

- α is the probability of making a Type I error
  - $1 - \alpha$ is the probability of not making a Type I error

Type II Error : Failing to reject H_0 when it is false

- β is the probability of making a Type II error
  - $1 - \beta$ is the probability of not making a Type II error

State of Nature

Conclusion	H_0 True	H_0 False
Reject H_0	α	$1 - \alpha$
Do not Reject H_0	$1 - \beta$	β

Typical Values

- Usually set α to a low value
  - So that there is only a small chance of rejecting a true H_0
  - Typically, $\alpha = 0.05$
    - For $\alpha = 0.05$, strong evidence is required to reject H_0
    - Usually choose α between 0.01 and 0.05
      - $\alpha = 0.01$ requires very strong evidence is to reject H_0
    - Sometimes choose α as high as 0.10
  - Tradeoff between α and β
    - For fixed sample size, the lower we set α, the higher β
      - And the higher α, the lower β

z Tests about a Population Mean
(σ Known): One-Sided Alternatives

- Test hypotheses about a (normal) population mean using the normal distribution
  - Called *z* tests
  - Require that the true value of the population standard deviation σ is known
    - In most real-world situations, σ is not known
      - But often is estimated from *s* of a single sample
      - When σ is unknown, test hypotheses about a population mean using the *t* distribution
    - Here, assume that we know σ
- Also use a “rejection rule”

Steps in Testing a “Greater Than” Alternative

The steps are as follows:

1. State the null and alternative hypotheses
2. Specify the significance level α
3. Select the test statistic
4. Determine the rejection rule for deciding whether or not to reject H_0
5. Collect the sample data and calculate the value of the test statistic
6. Decide whether to reject H_0 by using the test statistic and the rejection rule
7. Interpret the statistical results in managerial terms and assess their practical importance

Steps in Testing a “Greater Than”
Alternative in Trash Bag Case

1. State the null and alternative hypotheses
   - ❖ $H_0: \mu \leq 50$
     $H_a: \mu > 50$

     where μ is the mean breaking strength of the new bag
2. Specify the significance level α
   - ❖ $\alpha = 0.05$

Steps in Testing a "Greater Than" Alternative in Trash Bag Case (Cont.)

3. Select the test statistic
   - Use the test statistic

$$z = \frac{\bar{x} - 50}{\sigma_{\bar{x}}} = \frac{\bar{x} - 50}{\sigma / \sqrt{n}}$$

   - A positive value of this this test statistic results from a sample mean that is greater than 50 lbs
     - Which provides evidence against H_0 in favor of H_a

Steps in Testing a "Greater Than" Alternative in Trash Bag Case (Cont.)

4. Determine the rejection rule for deciding whether or not to reject H_0
   - ❖ To decide how large the test statistic must be to reject H_0 by setting the probability of a Type I error to α, do the following:
   - ❖ The probability α is the area in the right-hand tail of the standard normal curve
   - ❖ Use the normal table to find the point z_α (called the rejection or critical point)
     - ❖ z_α is the point under the standard normal curve that gives a right-hand tail area equal to α
     - ❖ Since $\alpha = 0.05$ in the trash bag case, the rejection point is $z_\alpha = z_{0.05} = 1.645$

Steps in Testing a “Greater Than”
Alternative in Trash Bag Case (Cont.)

4. Continued

- Reject H_0 in favor of H_a if the test statistic z is greater than the rejection point z_α
  - This is the rejection rule
- In the trash bag case, the rejection rule is to reject H_0 if the calculated test statistic z is > 1.645

Steps in Testing a “Greater Than”
Alternative in Trash Bag Case (Cont.)

5. Collect the sample data and calculate the value of the test statistic
   - ❖ In the trash bag case, assume that σ is known and σ = 1.65 lbs
   - ❖ For a sample of *n* = 40, $\bar{x}$ = 50.575 lbs. Then

$$z = \frac{\bar{x} - 50}{\sigma/\sqrt{n}} = \frac{50.575 - 50}{1.65/\sqrt{40}} = 2.20$$

Steps in Testing a "Greater Than"
Alternative in Trash Bag Case #6

6. Decide whether to reject H_0 by using the test statistic and the rejection rule
   - ❖ Compare the value of the test statistic to the rejection point according to the rejection rule
   - ❖ In the trash bag case, $z = 2.20$ is $> z_{0.05} = 1.645$
   - ❖ Therefore reject H_0: $\mu \leq 50$ in favor of H_a: $\mu > 50$ at the 0.05 significance level
     - ❖ Have rejected H_0 by using a test that allows only a 5% chance of wrongly rejecting H_0
     - ❖ This result is "statistically significant" at the 0.05 level

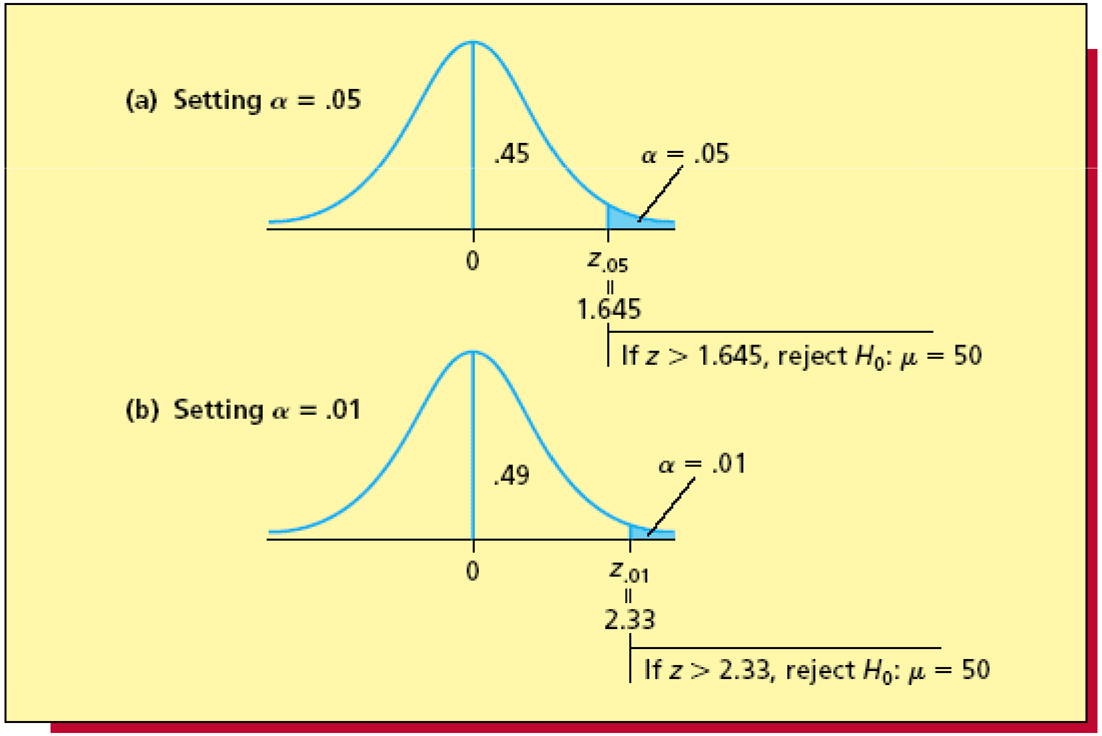
Steps in Testing a "Greater Than"
Alternative in Trash Bag Case #7

7. Interpret the statistical results in managerial terms and assess their practical importance
   - Can conclude that the mean breaking strength of the new bag exceeds 50 lbs

Steps in Testing a "Greater Than" Alternative in Trash Bag Case (Cont.)

Trash Bag Case:

Testing H_0: $\mu \leq 50$ versus H_a: $\mu > 50$ for:



(a) $\alpha = 0.05$

(b) $\alpha = 0.01$

Effect of α

- At $\alpha = 0.01$, the rejection point is $z_{0.01} = 2.33$
- In the trash example, the test statistic $z = 2.20 < z_{0.01} = 2.33$
- Therefore, cannot reject H_0 in favor of H_a at the $\alpha = 0.01$ significance level
  - This is the opposite conclusion reached with $\alpha = 0.05$
  - So, the smaller we set α, the larger is the rejection point, and the stronger is the statistical evidence that is required to reject the null hypothesis H_0

The p-Value

- The *p-value* or the *observed level of significance* is the probability of the obtaining the sample results if the null hypothesis H_0 is true
  - The p-value is used to measure the weight of the evidence against the null hypothesis
- Sample results that are not likely if H_0 is true have a low p-value and are evidence that H_0 is not true
  - The p-value is the smallest value of α for which we can reject H_0
- Use the p-value as an alternative to testing with a z test statistic

Steps Using a p-value to Test a
"Greater Than" Alternative

(Steps 1-3 are identical)

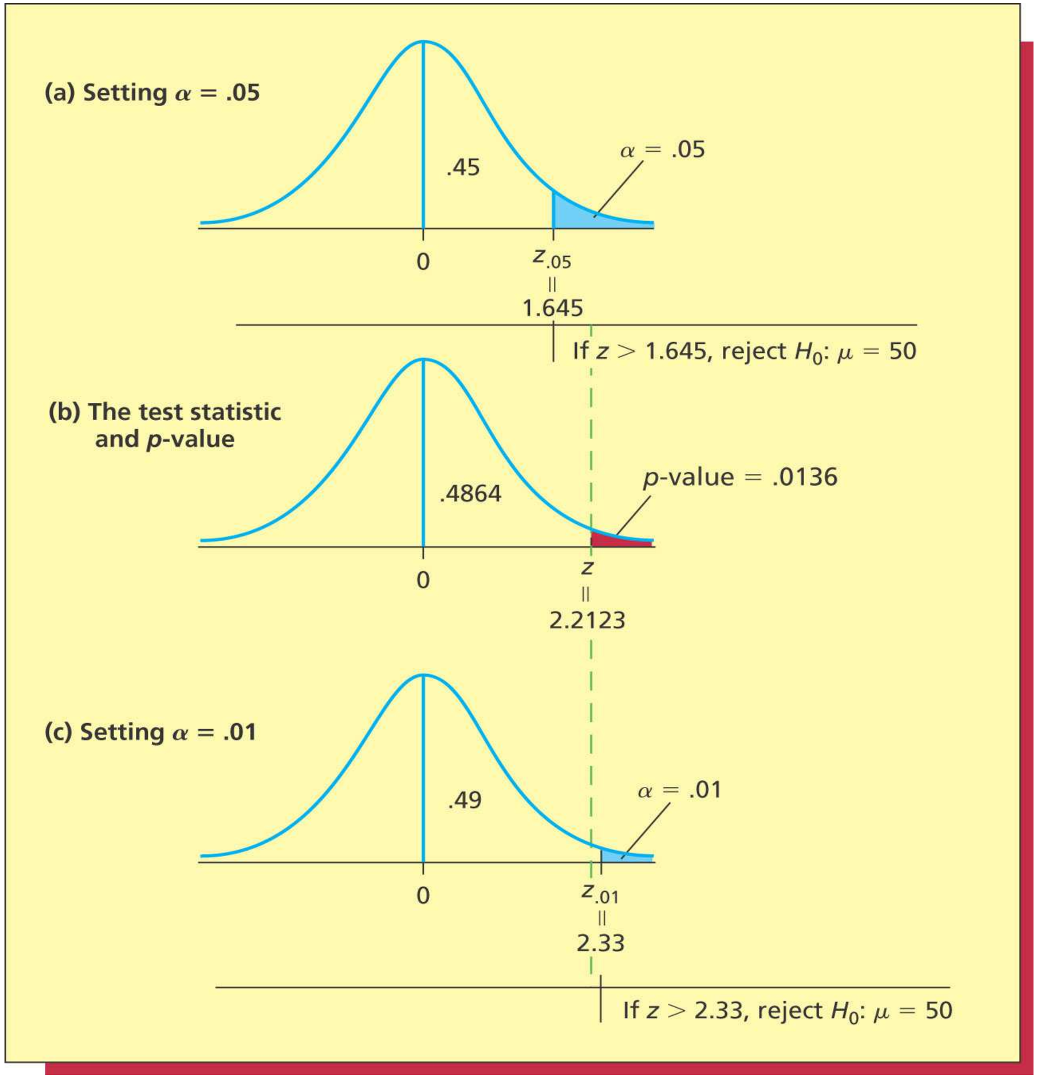
4. Collect the sample data and compute the value of the test statistic
   - ❖ In the trash bag case, the value of the test statistic was calculated to be $z = 2.20$
5. Calculate the p-value by corresponding to the test statistic value
   - ❖ In the trash bag case, the area under the standard normal curve in the right-hand tail to the right of the test statistic value $z = 2.20$
   - ❖ The area is 0.5 – 0.4861 = 0.0139
   - ❖ The p-value is 0.0139

Steps Using a p-value to Test a "Greater Than" Alternative (Cont.)

5. Continued
   - If H_0 is true, the probability is 0.0139 of obtaining a sample whose mean is 50.575 lbs or higher
   - This is so low as to be evidence that H_0 is false and should be rejected
6. Reject H_0 if the p-value is less than α
   - In the trash bag case, α was set to 0.05
   - The calculated p-value of 0.0139 is **<** $\alpha = 0.05$
     - This implies that the test statistic $z = 2.20$ is **>** the rejection point $z_{0.05} = 1.645$
   - Therefore reject H_0 at the $\alpha = 0.05$ significance level

Using a p-value to Test a
"Greater Than" Alternative

Testing H_0: $\mu \leq 50$ versus H_a: $\mu > 50$ by using rejection points and the *p*-value.



Steps in Testing a "Less Than" Alternative in Payment Time Case

1. State the null and alternative hypotheses
   - ❖In the payment time case, H_0: $\mu \geq 19.5$ vs. H_a: $\mu < 19.5$, where μ is the mean bill payment time (in days)
2. Specify the significance level α
   - ❖In the payment time case, set $\alpha = 0.01$

Steps in Testing a “Less Than”
Alternative in Payment Time Case (Cont.)

3. Select the test statistic

- In the payment time case, use the test statistic

$$z = \frac{\bar{x} - 19.5}{\sigma_{\bar{x}}} = \frac{\bar{x} - 19.5}{\sigma / \sqrt{n}}$$

- A negative value of this this test statistic results from a sample mean that is less than 19.5 days
  - Which provides evidence against H_0 in favor of H_a

Steps in Testing a “Less Than” Alternative in Payment Time Case (Cont.)

4. Determine the rejection rule for deciding whether or not to reject H_0
   - ❖ To decide how much less than 0 the test statistic must be to reject H_0 by setting the probability of a Type I error to α, do the following:
   - ❖ The probability α is the area in the left-hand tail of the standard normal curve
   - ❖ Use normal table to find the rejection point $-z_\alpha$
     - ❖ $-z_\alpha$ is the negative of z_α
     - ❖ $-z_\alpha$ is the point on the horizontal axis under the standard normal curve that gives a left-hand tail area equal to α

Steps in Testing a "Less Than"
Alternative in Payment Time Case (Cont.)

4. Continued
   - Because $\alpha = 0.01$ in the payment time case, the rejection point is $-z_{\alpha} = -z_{0.01} = -2.33$
   - Reject H_0 in favor of H_a if the test statistic z is calculated to be less than the rejection point $-z_{\alpha}$
     - This is the rejection rule
   - In the payment time case, the rejection rule is to reject H_0 if the calculated test statistic $-z$ is less than -2.33

Steps in Testing a "Less Than"
Alternative in Payment Time Case (Cont.)

5. Collect the sample data and calculate the value of the test statistic
   - In the payment time case, assume that σ is known and σ = 4.2 days
   - For a sample of *n* = 65, ☒ = 18.1077 days:

$$z = \frac{\bar{x} - 19.5}{\sigma / \sqrt{n}} = \frac{18.1077 - 19.5}{4.2/\sqrt{65}} = -2.67$$

Steps in Testing a "Less Than"
Alternative in Payment Time Case (Cont.)

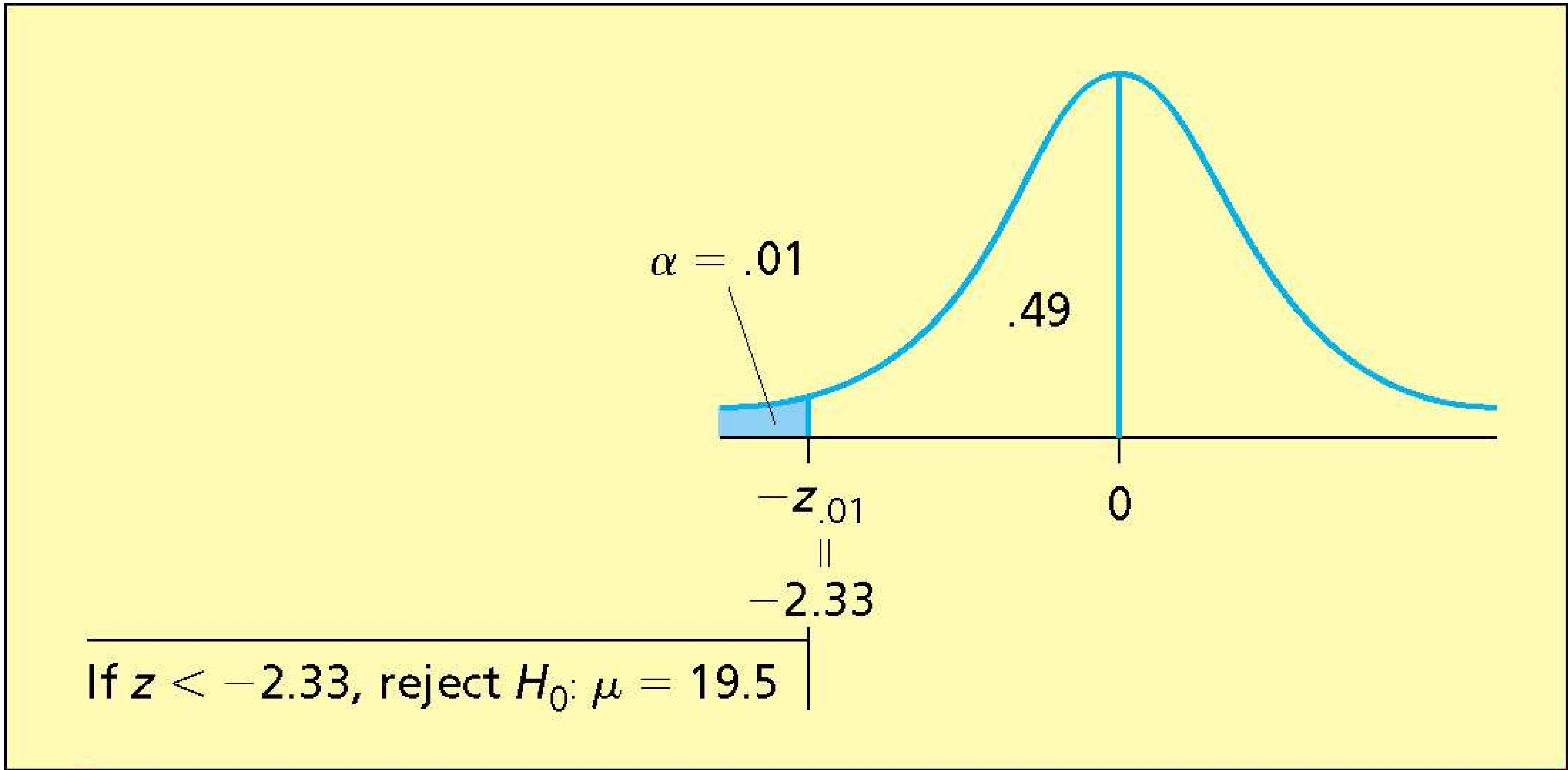
6. Decide whether to reject H_0 by using the test statistic and the rejection rule
   - ❖Compare the value of the test statistic to the rejection point according to the rejection rule
   - ❖In the payment time case, $z = –2.67$ is less than $z_{0.01} = –2.33$
   - ❖Therefore reject H_0: $\mu \geq 19.5$ in favor of H_a: $\mu < 19.5$ at the 0.01 significance level
     - ❖Have rejected H_0 by using a test that allows only a 1% chance of wrongly rejecting H_0
     - ❖This result is "statistically significant" at the 0.01 level

Steps in Testing a “Less Than”
Alternative in Payment Time Case (Cont.)

7. Interpret the statistical results in managerial terms and assess their practical importance
   - Can conclude that the mean bill payment time of the new billing system is less than 19.5 days

Testing a “Less Than”
Alternative in Payment Time Case (Cont.)

Testimi i H_0: $\mu \geq 19.5$ kundrejt H_a: $\mu < 19.5$ për $\alpha = 0.01$



Steps Using a p-value to Test a
"Less Than" Alternative

(Steps 1–3 are the same)

4. Collect the sample data and compute the value of the test statistic
   - In the payment time case, the value of the test statistic was calculated to be $z = -2.67$
5. Calculate the p-value by corresponding to the test statistic value
   - In the payment time case, the area under the standard normal curve in the left-hand tail to the left of the test statistic $z = -2.67$
   - The area is $0.5 - 0.4962 = 0.0038$
   - The p-value is 0.0038

Steps Using a p-value to Test a "Less Than" Alternative Continued

5. Continued
   - If H_0 is true, then the probability is 0.0038 of obtaining a sample whose mean is as low as 18.1077 days or lower
   - This is so low as to be evidence that H_0 is false and should be rejected
6. Reject H_0 if the p-value is less than α
   - In the payment time case, α was 0.01
   - The calculated p-value of 0.0038 is **<** $\alpha = 0.01$
     - This implies that the test statistic $z = -2.67$ is less than the rejection point $-z_{0.01} = -2.33$
   - Therefore, reject H_0 at the $\alpha = 0.01$ significance level

z Tests about a Population Mean
(σ Known): Two-Sided Alternatives

- Testing a "not equal to" alternative hypothesis
  - The sampling distribution of all possible sample means is modeled by a normal curve
  - Require that the true value of the population standard deviation σ is known
  - What's new and different here?
  - The area corresponding to the significance level α is evenly split into both the right- and left-hand tails of the standard normal curve

Steps in Testing a "Not Equal To"
Alternative in Camshaft Case (Cont.)

1. State the null and alternative hypotheses
   - In the camshaft case, H_0: $\mu = 4.5$ vs. H_a: $\mu \neq 4.5$, where μ is the mean hardness depth of the camshaft (in mm)
2. Specify the significance level α
   - In the camshaft case, set $\alpha = 0.05$
3. Select the test statistic
   - In the camshaft case, use the test statistic

$$z = \frac{\bar{x} - 4.5}{\sigma_{\bar{x}}} = \frac{\bar{x} - 4.5}{\sigma / \sqrt{n}}$$

Steps in Testing a “Not Equal To”

Alternative in Camshaft Case (Cont.)

3. Continued
   - A positive value of this test statistic results from a sample mean that is greater than 4.5 mm
     - Which provides evidence against H_0 and for H_a
   - A negative value of this test statistic results from a sample mean that is less than 4.5 mm
     - Which provides evidence against H_0 and for H_a
   - A very small value close to 0 (either very slightly positive or very slightly negative) of this test statistic results from a sample mean that is nearly 4.5 mm
     - Which provides evidence in favor of H_0 and against H_a

Steps in Testing a "Not Equal To"
Alternative in Camshaft Case (Cont.)

4. Determine the rejection rule for deciding whether or not to reject H_0
   - ❖To decide how different the test statistic must be from zero (positive or negative) to reject H_0 in favor of H_a by setting the probability of a Type I error to α, do the following:
   - ❖Divide the α in half to find $\alpha/2$; $\alpha/2$ is the tail area in both tails of the standard normal curve
     - ❖Under the standard normal curve, the probability $\alpha/2$ is the area in the right-hand tail and probability $\alpha/2$ is the area in the left-hand tail

Steps in Testing a "Not Equal To"
Alternative in Camshaft Case (Cont.)

4. Continued
   - ❖ Use the normal table to find the rejection points $z_{\alpha/2}$ and $-z_{\alpha/2}$
     - ❖ $z_{\alpha/2}$ is the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to $\alpha/2$
     - ❖ $-z_{\alpha/2}$ is the point on the horizontal axis under the standard normal curve that gives a left-hand tail area equal to $\alpha/2$

Steps in Testing a "Not Equal To"
Alternative in Camshaft Case (Cont.)

4. Continued
   - ❖ Because $\alpha = 0.05$ in the camshaft case, $\alpha/2=0.025$
     - ❖ The area under the standard normal to the right of the mean is 0.5 – 0.025 = 0.475
     - ❖ From Table A.3, the area is 0.475 for z = 1.96
   - ❖ The rejection points are

$$z_\alpha = z_{0.025} = 1.96 \quad \text{OR} \quad -z_\alpha = -\,z_{0.025} = -\,1.96$$

   - ❖ Reject H_0 in favor of H_a if the test statistic z satisfies either:

     z greater than the rejection point $z_{\alpha/2}$

     OR

     $-z$ less than the rejection point $-z_{\alpha/2}$

     - ❖ This is the rejection rule

Steps in Testing a “Not Equal To”
Alternative in Camshaft Case (Cont.)

5. Collect the sample data and calculate the value of the test statistic
   - In the camshaft case, assume that σ is known and σ = 0.47 mm
   - For a sample of *n* = 40, ⊠ = 4.26 mm
   - Then

$$z = \frac{\bar{x} - 4.5}{\sigma / \sqrt{n}} = \frac{4.26 - 4.5}{0.47 / \sqrt{35}} = -3.02$$

Steps in Testing a "Not Equal To"
Alternative in Camshaft Case (Cont.)

6. Decide whether to reject H_0 by using the test statistic and the rejection rule
   - ❖ Compare the value of the test statistic to the rejection point according to the rejection rule
   - ❖ In the camshaft case, $-z = -3.02$ is **<** $-z_{0.025} = -1.96$
   - ❖ Therefore reject H_0: $\mu = 4.5$ in favor of H_a: $\mu \neq 4.5$ at the 0.05 significance level
     - ❖ Have rejected H_0 by using a test that allows only a 5% chance of wrongly rejecting H_0
     - ❖ This result is "statistically significant" at the 0.05 level

Steps in Testing a “Not Equal To”
Alternative in Camshaft Case (Cont.)

7. Interpret the statistical results in managerial terms and assess their practical importance
   - Can conclude that the mean hardness depth of the camshaft is not 4.5 mm

Steps Using a p-value to Test a "Not Equal To" Alternative

(Steps 1–3 are the same)

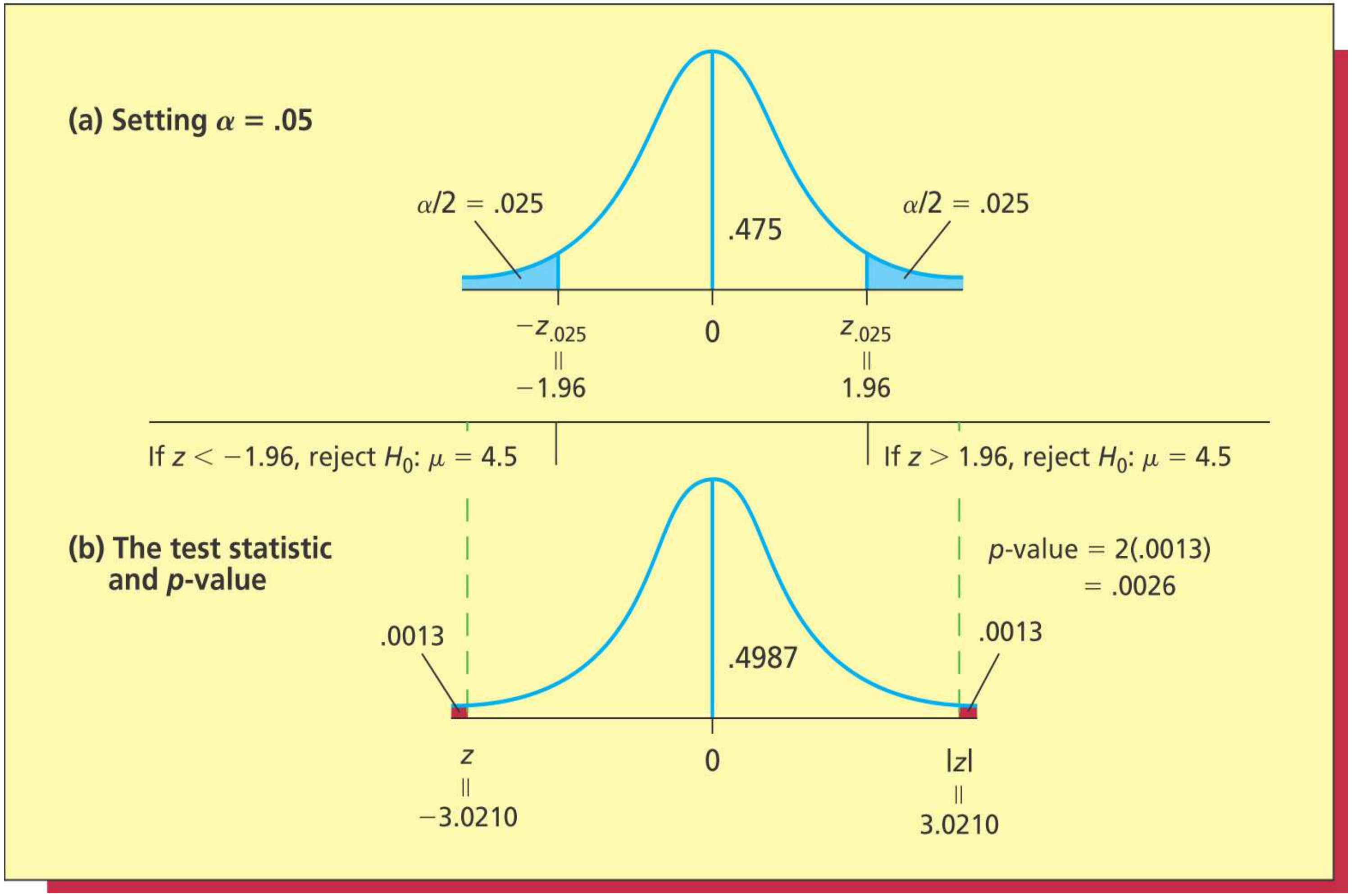
4. Collect the sample data and compute the value of the test statistic
   - ❖ In the camshaft case, the value of the test statistic was calculated to be $z = -3.02$
5. Calculate the p-value by corresponding to the test statistic value
   - ❖ In the camshaft case, the area under the standard normal curve in the left-hand tail to the left of the test statistic value $z = -3.02$
   - ❖ The area is 0.5 – 0.4987 = 0.0013
   - ❖ The p-value is 2(0.0013) = 0.0026

Steps Using a p-value to Test a
"Not Equal To" Alternative (Cont.)

5. Continued
   - That is, if H_0 is true, then the probability is 0.0013 of obtaining a sample whose mean is as small as 4.26 mm or less
   - This probability is so low as to be evidence that H_0 is false and should be rejected
6. Reject H_0 if the p-value is less than α
   - In the camshaft case, α was 0.05
   - The calculated p-value of $0.0026 < \alpha = 0.05$
   - Therefore reject H_0 at the $\alpha = 0.05$ significance level

Testing a “Not Equal To” Alternative

Testing H_0: $\mu = 4.5$ versus H_a: $\mu \neq 4.5$ for $\alpha = 0.05$



Small Sample Tests

about a Population Mean

If the sampled population is normal, we can **reject $H_0: \mu = \mu_0$ at the α level of significance** (probability of Type I error equal to α) if and only if the appropriate **rejection point condition** holds or, equivalently, if the corresponding p-value is less than α.

Alternative	Reject H_0 if:	p-Value
$H_a: \mu > \mu_0$	$z > z_\alpha$	Area under std normal curve right of z
$H_a: \mu < \mu_0$	$z < -z_\alpha$	Area under std normal curve left of z
$H_a: \mu \neq \mu_0$	$\lvert z \rvert > z_{\alpha/2}$, that is $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$	Twice area under std normal curve right of $\lvert z \rvert$

Test Statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

If σ unknown and n is large, estimate σ by s.

Weight of Evidence Against the Null

- Calculate the test statistic and the corresponding p-value
- Rate the strength of the conclusion about the null hypothesis H_0 according to these rules:
  - If $p < 0.10$, then there is **some** evidence to reject H_0
  - If $p < 0.05$, then there is **strong** evidence to reject H_0
  - If $p < 0.01$, then there is **very strong** evidence to reject H_0
  - If $p < 0.001$, then there is **extremely strong** evidence to reject H_0

Confidence Intervals vs.
Hypothesis Testing

- The null hypothesis H_0 can be rejected in favor of the alternative H_a by setting the probability of a Type I error equal to α if and only if the $100(1 – \alpha)\%$ confidence interval for μ does not contain μ_0
  - where μ_0 is the claimed value for the population mean
- In other words, if the confidence interval does not contain the claimed value μ_0, then it can be rejected as being false

Confidence Intervals vs.
Hypothesis Testing Continued

- The α used here is the same α used in Chapter 7
  - The confidence level is $1 - \alpha$
  - The significance level is α
  - $(1 - \alpha) + \alpha = 1$, so the confidence level and significance levels are complementary

t Tests about a Population Mean
(σ Unknown)

- Suppose the population being sampled is normally distributed
- The population standard deviation σ is unknown, as is the usual situation
  - If the population standard deviation σ is unknown, then it will have to estimated from a sample standard deviation *s*
- Under these two conditions, have to use the *t* distribution to test hypotheses
  - The *t* distribution from Chapter 7

Defining the *t* Random Variable
(σ Unknown)

- Define a new random variable *t*:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

  - with the definition of symbols as before
- The sampling distribution of this random variable is a *t* distribution with $n - 1$ degrees of freedom

Defining the *t* Statistic

(σ Unknown)

- Let ☒ be the mean of a sample of size *n* with standard deviation *s*
- Also, μ_0 is the claimed value of the population mean
- Define a new test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- If the population being sampled is normal, and
- If *s* is used to estimate σ, then …
- The sampling distribution of the *t* statistic is a *t* distribution with *n* – 1 degrees of freedom

t Tests about a Population Mean
(σ Unknown)

- Reject H_0: $\mu = \mu_0$ in favor of a particular alternative hypothesis H_a at the α level of significance if and only if the appropriate rejection point rule or, equivalently, the corresponding p-value is less than α
- We have the following rules …

t Tests about a Population Mean
(σ Unknown) (Cont.)

Alternative	**Reject H_0 if:**	**p-value**
$H_a: \mu > \mu_0$	$t > t_\alpha$	Area under t distribution to right of t
$H_a: \mu < \mu_0$	$t < -t_\alpha$	Area under t distribution to left of –t
$H_a: \mu \neq \mu_0$	$\lvert t \rvert > t_{\alpha/2}$ *	Twice area under t distribution to right of $\lvert t \rvert$

- t_α, $t_{\alpha/2}$, and p-values are based on $n - 1$ degrees of freedom (for a sample of size n)

* either $t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$

t-testet mbi një mesatare popullimi (σ e panjohur). (Vazhdim)

Reject H_0: $\mu = \mu_0$ in favor of a particular alternative hypothesis H_a at the α level of significance if and only if the appropriate rejection point rule or, equivalently, the corresponding p-value is less than α

Alternative	**Reject H_0 if:**	**p-Value**
$H_a: \mu > \mu_0$	$t > t_\alpha$	Area under t distribution on right of t
$H_a: \mu < \mu_0$	$t < -t_\alpha$	Area under t distribution on left of t
$H_a: \mu \neq \mu_0$	$\lvert t \rvert > t_{\alpha/2}$, that is $t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$	Twice area under t distribution on right of $\lvert t \rvert$

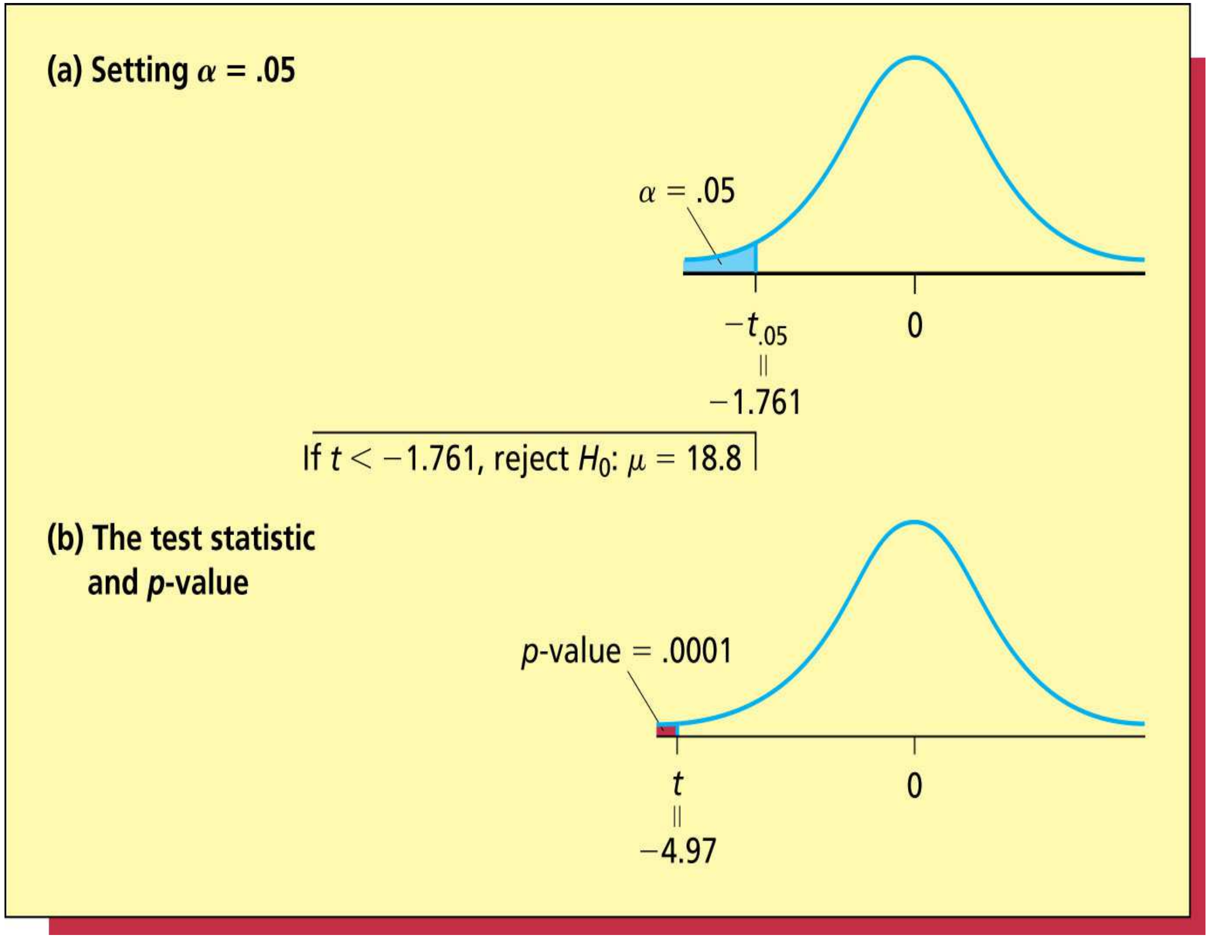
Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

t_α, $t_{\alpha/2}$ and p-values are based on n – 1 degrees of freedom.

Example: Small Sample Test
about a Mean

Example 8.4. testing H_0: $\mu \geq 18.8$ versus H_a: $\mu < 18.8$ for $\alpha = 0.05$



Hypothesis Tests about a Population Proportion

- Reject H_0: $p = p_0$ at the α level of significance (probability of Type I error equal to α) if and only if the appropriate rejection point condition holds or, equivalently, if the corresponding p-value is less than α
- We have the following rules …

Hypothesis Tests about a Population Proportion Continued

Alternative	Reject H_0 if:	p-value
$H_a: p > p_0$	$z > z_{\alpha}$	Area under standard normal to the right of z
$H_a: p < p_0$	$z < -z_{\alpha}$	Area under standard normal to the left of $-z$
$H_a: p \neq p_0$	$\lvert z \rvert > z_{\alpha/2}$ *	Twice the area under standard normal to the right of $\lvert z \rvert$

where the test statistic is:

$$z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

* either $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

Hypothesis Tests about a Population Proportion

If the sample size n is large, we can **reject H_0: $p = p_0$ at the α level of significance** (probability of Type I error equal to α) if and only if the appropriate **rejection point condition** holds or, equivalently, if the corresponding p-value is less than α.

Alternative	Reject H_0 if:	p-Value
$H_a: p > p_0$	$z > z_\alpha$	Area under std normal curve right of z
$H_a: p < p_0$	$z < -z_\alpha$	Area under std normal curve left of z
$H_a: p \neq p_0$	$\lvert z \rvert > z_{\alpha/2}$, that is $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$	Twice area under std normal curve right of $\lvert z \rvert$

Test Statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Example: Hypothesis Tests about a Proportion

- Example 8.7.
  - Testing H_0: $p \le 0.70$ versus H_a: $p > 0.70$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.77 - 0.70}{\sqrt{\frac{0.70(1-0.70)}{300}}} = 2.65$$

$$z = 2.65 > z_{0.05} = 1.645, \quad z = 2.65 > z_{0.01} = 2.33, \quad z = 2.65 < z_{0.001} = 3.09$$

$$p\text{-value} = P(z \ge 2.65) = (0.5 - 0.4960) = 0.004$$

Type II Error Probabilities

- Want the probability β of not rejecting a false null hypothesis
  - That is, want the probability β of committing a Type II error
  - $1 - \beta$ is called the power of the test

Calculating β

- Assume that the sampled population is normally distributed, or that a large sample is taken
- Test H_0: $\mu = \mu_0$ vs H_a: $\mu < \mu_0$ or H_a: $\mu > \mu_0$ or H_a: $\mu \neq \mu_0$
- Want to make the probability of a Type I error equal to α and randomly select a sample of size *n*
- The probability β of a Type II error corresponding to the alternative value μ_a for μ is equal to the area under the standard normal curve to the left of

$$z^* - \frac{\left|\mu_0 - \mu_a\right|}{\sigma / \sqrt{n}}$$

  Here z^* equals z_α if the alternative hypothesis is one-sided ($\mu < \mu_0$ or $\mu > \mu_0$)
- Also $z^* \neq z_{\alpha/2}$ if the alternative hypothesis is two-sided ($\mu \neq \mu_0$)

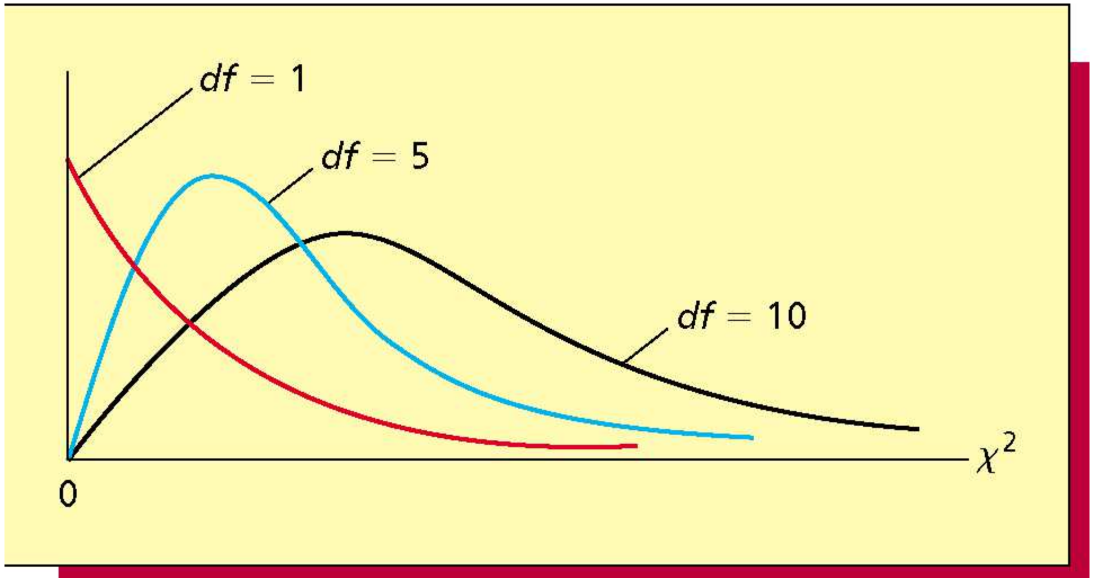
Sample Size

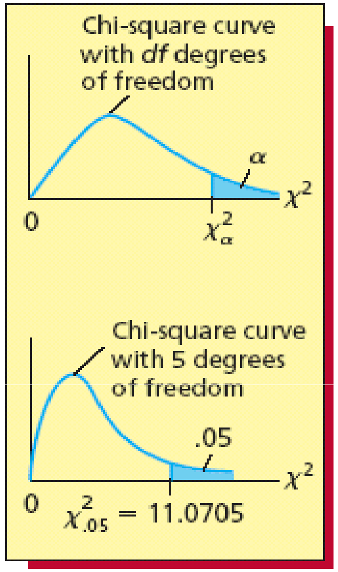
- Assume that the sampled population is normally distributed, or that a large sample is taken
- Test H_0: $\mu = \mu_0$ vs. H_a: $\mu < \mu_0$ or H_a: $\mu > \mu_0$ or H_a: $\mu \neq \mu_0$
- Want to make the probability of a Type I error equal to α and the probability of a Type II error corresponding to the alternative value μ_a for μ equal to β
- Then take a sample of size:

$$n = \frac{\left(z^* + z_\beta\right)^2 \sigma^2}{\left(\mu_0 - \mu_a\right)^2}$$

- Here z^* equals z_α, if the alternative hypothesis is one-sided ($\mu < \mu_0$ or $\mu > \mu_0$) and z^* equals $z_{\alpha/2}$ if the alternative hypothesis is two-sided ($\mu \neq \mu_0$)
- Also z_β is the point on the scale of the standard normal curve that gives a right-hand tail area equal to β

The Chi-Square Distribution





The **chi-square χ^2 distribution** depends on the number of **degrees of freedom**

- See Table A.17

A **chi-square point** χ^2_α is the point under a chi-square distribution that gives right-hand tail area α

Statistical Inference for
Population Variance

If s^2 is the variance of a random sample of *n* measurements from a normal population with variance σ^2, then the sampling distribution of the statistic $(n - 1)\, s^2 / \sigma^2$ is a chi-square distribution with $(n - 1)$ degrees of freedom and

100(1-α)% confidence interval for σ^2

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \right)$$

All chi-square points are based on $n - 1$ degrees of freedom

Test of H_0: $\sigma^2 = \sigma^2_0$

Test Statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Reject H_0 in favor of

$$H_a : \sigma^2 > \sigma_0^2 \text{ if } \chi^2 > \chi^2_\alpha$$
$$H_a : \sigma^2 < \sigma_0^2 \text{ if } \chi^2 < \chi^2_{1-\alpha}$$
$$H_a : \sigma^2 \neq \sigma_0^2 \text{ if } \chi^2 > \chi^2_\alpha \text{ or } \chi^2 < \chi^2_{1-\alpha}$$

Summary: Selecting an Appropriate Test Statistic for a Test about a Population Mean