Chapter 4

Calculus of Several Variables

4.1 Functions of Several Variables

Exercises

1. Compute the indicated function values.

   (a) $f(x, y) = (x-1)^2 + 2xy^3$; $f(2, -1)$; $f(1, 2)$

   (b) $f(x, y) = \frac{3x+2y}{2x+3y}$; $f(1, 2)$; $f(-4, 6)$

   (c) $g(x, y) = \sqrt{y^2 - x^2}$; $g(4, 5)$; $g(-1, 2)$

   (d) $f(s, t) = \frac{e^{st}}{2-e^{st}}$; $f(1, 0)$; $f(\ln 2, 2)$

   (e) $f(x, y, z) = xyz$; $f(1, 2, 3)$; $f(3, 2, 1)$

2. describe the domain of the given function.

   (a) $f(x, y) = \frac{5x+2y}{4x+3y}$

   (b) $f(x, y) = \sqrt{9 - x^2 - y^2}$

(c) $f(x,y) = \sqrt{x^2 - y}$

(d) $f(x,y) = \frac{x}{\ln(x+y)}$

(e) $f(x,y) = \frac{e^{xy}}{\sqrt{x-2y}}$

3. Sketch the indicated level curve $f(x,y) = C$ for each choice of constant C.

   (a) $f(x,y) = x + 2y$; $C = 1$; $C = 2$; $C = -3$

   (b) $f(x,y) = x^2 + y$; $C = 0$; $C = 4$; $C = 9$

   (c) $f(x,y) = x^2 - 4x - y$; $C = -4$; $C = 5$

   (d) $f(x,y) = \frac{x}{y}$; $C = -2$; $C = 2$

   (e) $f(x,y) = xy$; $C = 1$; $C = -1$; $C = 2$; $C = -2$

   (f) $f(x,y) = xe^y$; $C = 1$; $C = e$

4. Using x skilled workers and y unskilled workers, a manufacturer can produce $Q(x,y) = 10x^2y$ units per day. Currently there are 20 skilled workers and 40 unskilled workers on the job.

   (a) How many units are currently being produced each day?

   (b) By how much will the daily production level change if 1 more skilled worker is added to the current workforce?

   (c) By how much will the daily production level change if 1 more unskilled worker is added to the current workforce?

   (d) By how much will the daily production level change if 1 more skilled worker and if 1 more unskilled worker are added to the current workforce?

5. A manufacturer can produce scientific graphing calculators at a cost of 40 € apiece and business calculators for 20 € apiece

   (a) Express the manufacturer's total monthly production cost as a function of the number of graphing calculators and the number of business calulators produced.

   (b) Compute the total monthly cost if 500 scientific and 800 business calculators are produced.

(c) The manufacutrer wants to increase the ouutput of scientific calculators by 50 a month from the level in part (b). What corresponding change should be made in the monthly output of business calculators so the total monthly cost will not change.

6. A paint store carries two brands of latex paint. Sales figures indicate that if the first brand is sold for x_1 euros per liter and the second for x_2 euros per litter, the demand for the first brand will be $D_1(x_1, x_2) = 200 - 10x_1 + 20x_2$ liters per month and the demand for the second brand will be $D_2(x_1, x_2) = 100 + 5x_1 - 10x_2$ liters per month

   (a) Express the paint store's total monthly revenue from the sale of the paint as a function of the prices x_1 and x_2.

   (b) Compute the revenue in part (a) if the first brand is sold for 6 € per liter and the second for 5 € per liter.

7. The output at a certain factory is $Q(K, L) = 120K^{2/3}L^{1/3}$ units, where K is the capital investment meassured in units of 1,000 € and L is the size on the labor force meassured in worker-hours.

   (a) Compute the output if the capital investment is 125,000 € and the size of labor force is 1,331 worker-hours.

   (b) What will happen to the output in part (a) if both the level of capital investment and the size of the labor force are cut in half?

8. Using x skilled and y unskilled workers, a manufacturer can produce $Q(x, y) = 3x + 2y$ units per day. Currently the workforce consists of 10 skilled workers and 20 unskilled workers.

   (a) Compute the current daily output.

   (b) Find an equation relating the levels of skilled and unskilled labor if the daily output is to remain at its current level.

   (c) On a two-dimensional coordinate system, draw the isoquant (constant production curve) thet corresponds to the current level of output.

(d) What change should be maid in the level of unskilled labor y to offset an increase in skilled labor x of two workers so that the output will remain at its current level?

9. Te utility derived by a consumer from x units of one commodity and y units of a second is given by the utility function $U(x,y) = 2x^3y^2$. Te consumer currently owns $x = 5$ units of the first commodity and $y = 4$ units of the second commodity. Find the consumer's current level of utility and sketch the corresponding indfference curve.

10. Te utility derived by a consumer from x units of one commodity and y units of a second is given by the utility function $U(x,y) = (x+1)(y+2)$. Te consumer currently owns $x = 25$ units of the first commodity and $y = 8$ units of the second. Find the consumer's current level of utility and sketch the corresponding indfference curve.