

# Chapter 1

## Integration

### 1.1 Antidifferentiation: The Indefinite Integral

#### Exercises

1. Find the following integrals:

(a)  $\int \sqrt[3]{x} \, dx;$

(b)  $\int \frac{1}{\sqrt{x}} \, dx;$

(c)  $\int e^{-3t} \, dt.$

2. Find the following integrals:

(a)  $\int (x^3 - 3x^2 + 5) \, dx;$

(b)  $\int (2e^{-x} - \sqrt{x}) \, dx;$

(c)  $\int \left( 2e^u + \frac{6}{u} + \ln 2 \right) \, du;$

(d)  $\int \frac{y^2 + 3y - 2}{\sqrt{y}} \, dy;$

(e)  $\int \sqrt{t}(t^2 - 1) \, dt.$

3. The promoters of a fair estimate that  $t$  hours after the gates open at 9:00, visitors will be entering the fair at the rate of  $265 + 210t - 15t^2$  people per hour. Find the number of people who will enter the fair between 11:00 and 13:00.
4. The promoters of an exhibition estimate that  $t$  hours after the gates open at 9:00, visitors will be visiting the exhibition at the rate of  $N'(t)$  people per hour. Find an expression for the number of people who will visit the exhibition between 12:00 and 15:00.
5. It is estimated that  $t$  years from now the value of a certain parcel of land will be increasing at the rate of  $V'(t)$  euros per year. Find an expression for the amount by which the value of land will increase during the next 5 years.
6. A retailer receives a shipment of 10,000 kilograms of rice, that will be used up at the constant rate of 2,000 kilograms per month. If storage costs are 1 cent per kilogram per month, how much will the retailer pay in storage costs over the next 5 months?
7. A manufacturer has found that marginal cost is  $6x + 1$  euros per unit when  $x$  units have been produced. The total cost of producing the first unit is 130 €. What is the total cost of producing the first 10 units?
8. The marginal profit of a certain company is  $100 - 2q$  euros per unit when  $q$  units are produced. If the company's profit is 700 € when 10 units are produced, what is the company's maximum possible profit?
9. A manufacturer estimates marginal revenue to be  $\frac{100}{\sqrt{q}}$  euros per unit when the level of production is  $q$  units. The corresponding marginal cost has been found to be  $0.4q$  euros per unit. Suppose the manufacturer's profit is 520 € when the level of production is 16 units. What is the manufacturer's profit when the level of production is 25 units?
10. Prove the following rules for indefinite integrals:

(a) The constant rule:  $\int k \, dx = kx + C$

- (b) The logarithmic rule:  $\int \frac{dx}{x} = \ln |x| + C$   
(Hint: Consider the cases when  $x > 0$ , i.e.  $|x| = x$ , and when  $x < 0$ , i.e.  $|x| = -x$ .)
- (c) The exponential rule:  $\int e^{kx} dx = \frac{1}{k} e^{kx} + C$

11. Prove the following algebraic rules for indefinite integrals:

- (a) The sum rule:  $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$
- (b) The difference rule:  $\int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx$