2.3 The Product and Quotient Rules

Exercises

1. Differentiate the following functions:
   (a) $f(x) = x^2 e^x$;
   (b) $f(x) = xe^{-x}$;
   (c) $f(x) = x - \ln x$;
   (d) $f(x) = \ln x^3$;
   (e) $f(x) = \ln 2x$;
   (f) $f(x) = x^2 \ln x$;
   (g) $f(x) = x \ln \sqrt{x}$;
   (h) $f(x) = x^2 \ln \sqrt{x}$;
   (i) $f(x) = \frac{\ln x}{x}$.

2. Suppose that the revenue function for a product is given by

$$R(x) = 10x + \frac{100x}{3x+5}$$

   euros, where x is the number of units sold. At what rate does the revenue change with respect to the number of units sold when 15 units are produced.

3. If the cost of removing p percent of the particulate pollution from the exhaust gases at an industrial site si given by

$$C(p) = \frac{8{,}100p}{100-p}$$

   euros, find the rate of change of C with respect to p.

4. A travel agency will plan a group tour for groups of size 25 or larger. If the group contains exactly 25 people, the cost is 300 € per person. Each person's cost is reduced by 10 € for each additional person above the 25. Find the rate of change of revenue with respect to the number of people if the group contains 30 people.

5. The sales of a product is related to advertising expenses (in thousands of euros) by
$$s(x) = \frac{200x}{x+10}$$
thousand euros. Find the rate of change of sales with respect to advertising expenses when
   (a) $x = 10$;
   (b) $x = 20$.
6. A certain industrial machine depreciates so that its value after t years becomes $V(t) = 20{,}000e^{-t}$ euro.
   (a) At what rate is the value of the machine changing with respect to time after 5 years?
   (b) At what percentage rate is the value of the machine changing with respect to time after t years? Does this percentage rate depend on t or is it constant?
7. The consumer demand for a certain commodity is $D(p) = 3{,}000e^{-p}$ units per month when the market price is p hundred euros per unit. Express the consumers' total monthly expenditure $E(p)$ for the commodity as a function of p. What happens to the consumers' total monthly expenditure when the market price is 95 €? What happens when the price is 105 €? What can you conclude about the consumer's total monthly expenditure when the price is 100 €?
8. The number of years t that it takes for an investment to double is a function of the interest rate r, compounded continuously, according to
$$t(r) = \frac{\ln 2}{r}.$$
At what rate is the required time changing with respect to r if $r = 10\%$, compounded continuously.
9. After the end of an advertising campaign, the weekly sales of a certain product is given by
$$S(t) = 100{,}000e^{-0.5t}$$

units t weeks after the end of the campaign. Find the rate of change of weekly sales with respect to time (that is, the rate of *sales decay*).