

1.4 Functional Models

Exercises

1. The price of a certain commodity, when x units are sold, is $p = 50x + 20$ euros apiece. Express the revenue generated by selling the commodity as a function of x .
2. The rate at which people are implicated in a government scandal is directly proportional to both the number of people already implicated and the number of people involved who have not been yet implicated. Express this rate as a function of the number of people who have been implicated.
3. A truck is hired to transport goods from a factory to a warehouse. The driver's wages are figured by the hour and so are inversely proportional to the speed at which the truck is driven. The cost of fuel is directly proportional to the speed. Express the total cost of operating the truck as a function of the speed at which it is driven.
4. A bookstore can obtain a certain book from the publisher at a cost of 3 € per book. The bookstore has been offering the book at the price of 15 € per copy, and at this price, has been selling 200 copies a month. The bookstore is planning to lower its price to stimulate sales and estimates that for each 1 € reduction in the price, 20 more books will be sold each month.
 - (a) Express the bookstore's monthly profit from the sale of this book as a function of the selling price.
 - (b) Sketch the graph of the profit function. Estimate the optimal selling price.
5. A retailer can obtain cameras from the manufacturer at a cost of 50 € apiece. The retailer has been selling cameras at the price of 80 € apiece, and at this price, consumers have been buying 40 cameras a month. The retailer is planning to lower the price and estimates that for each 5 € reduction in the price, 10 more cameras will be sold each month. Express

the retailer's monthly profit from the sale of cameras as a function of the selling price. Draw the graph and estimate the optimal selling price.

6. A manufacturer can produce bookcases at a cost of 80 € apiece. Sales figures indicate that if the bookcases are sold for p euros apiece, approximately $500 - p$ will be sold each month. Express the manufacturer's monthly profit as a function of the selling price, draw the graph, and estimate the optimal selling price.
7. The supply and demand functions for a certain commodity are $S(p) = 4p + 200$ and $D(p) = -3p + 480$, respectively. Find the equilibrium price and the corresponding number of units supplied and demanded. Draw the supply and demand curves on the same set of axes.
8. The supply and demand functions for a certain commodity are $S(p) = p - 10$ and $D(p) = \frac{5,600}{p}$, respectively.
 - (a) Find the equilibrium price and the corresponding number of units supplied and demanded.
 - (b) Draw the supply and demand curves on the same set of axes.
 - (c) Where does the supply curve cross the p axes? Describe the economic significance of this point.
9. Suppose it is known that supply and demand for a certain commodity are functions of the form $S(p) = p^2 + Ap - 3$ and $D(p) = Cp + 32$, where p is the price per unit and A and B are constants. It is also known that no units will be supplied until the price is at least 3 € and that when the price exceeds 8 € per unit no items will be demanded. How many units are supplied and demanded at equilibrium? What is the excess of supply over demand when the price is 6 €?
10. A group of wholesalers will buy 50 dryers per month if the price is 200 € and 30 per month if the price is 300 €. The manufacturer is willing to supply 20 if the price is 210 € and 30 if the price is 230 €. Assuming that the resulting supply and demand functions are linear, find the equilibrium point for the market.

11. Retailers will buy 45 cordless phones from a wholesaler if the price is 10 € each and will buy 20 if the price is 60 €. The wholesaler will supply 35 phones at 30 € each and 70 at 50 € each. Assuming the supply and demand functions are linear, find the market equilibrium point.
12. A manufacturer of shower-sorrounds has a revenue function of $R(x) = 81.5x$ and a total cost function of $C(x) = 63x + 1,850$. Find the number of units that must be sold to break even.
13. A manufacturer sells belts for 12 € per unit. The fixed costs are 1,600 € per month, and the variable costs are 8 € per unit.
 - (a) Write the equations of the revenue and total cost functions.
 - (b) Find the break-even point.
14. A company manufactures and sells bookcases. The selling price is 54.90 € per bookcase. The total cost function is linear, and costs amount to 50,000 € for 2,000 bookcases and 32,120 € for 800 bookcases.
 - (a) Write the equations for the revenue.
 - (b) Write the equations for the total cost.
 - (c) Find the break-even point.
15. During a summer, a group of students builds motorcycles in a converted garage. The rental for the garage is 1,500 € for the summer, and the materials needed to build a motorcycle cost 125 €. The motorcycles can be sold for 275 € apiece.
 - (a) How many motorcycles must the students sell to break even?
 - (b) How many motorcycles must the students sell to make a profit 1,000 €?