1.2 The Graph of a Function

Exercises

In Exercises 1–6, sketch the graph. Include all x- and y-intercepts.

1. $f(x) = x$;
2. $f(x) = x^2$;
3. $f(x) = -x^2$;
4. $f(x) = x^3 - x^2 - 6x$;
5. $f(x) = x^4$;
6. $f(x) = \sqrt{x}$.

In Exercises 7–11, find the points of intersection (if any) of the given pair of graphs.

7. $y = 3x + 5$ and $y = -x + 3$;
8. $y = x^2$ and $y = 3x + 3$;
9. $y = -x^2$ and $y = x - 6$;
10. $y = x^3 - 6x^2$ and $y = -x^2$;
11. $y = x^2 - x$ and $y = 2 - x^2$.
12. A manufacturer can produce CD readers at a cost of 40 € apiece. It is estimated that if the tape recorders are sold for x euros apiece, consumers will bye $120 - x$ of them a month. Express the manufacturers monthly profit as a function of price, graph this function, and use the graph to estimate the optimal selling price.
13. The consumer demand for a certain commodity is $D(p) = -200p + 12,000$ units per month when the market price is p euros per unit.

(a) Graph the demand function.

(b) Express consumers' total monthly expenditure for the commodity as a function of p. (The total monthly expenditure is the total amount of money consumers spend each month on the commodity.)

(c) Graph the total monthly expenditure function.

(d) Use the graph to estimate the market price that generates the greatest consumer expenditure.

14. Suppose the total cost of manufacturing x units of a certain commodity is $C(x) = \frac{1}{6}x^3 + 2x + 5$ euros. Express the average cost per unit as a function of the number of units produced and, on the same set of axes, sketch the total cost and average cost functions.

15. A private university has launched a fundraising campaign. Suppose that college officials estimate that it will take $f(x) = \frac{10x}{150-x}$ weeks to reach $x\%$ of their goal.

    (a) Sketch the relevant portion of the graph of this function.

    (b) How long will it take to reach 50% of the campaign's goal?

    (c) How long will it take to reach 100% of the campaign's goal?

16. An advertising agency has found that when it promotes a new product in a city of 350,000 people, the rate of change of the number of people x who are aware of the product is given by the function

$$R(x) = 28{,}000 - 0.08x.$$

    Use the x- and y-intercepts to determine the relevant portion of the domain of this function and graph the function.

17. Because ot the resulting pollution by coal thermoelectric plants, the water streams in a certain region can't support fish. Suppose the cost of obtaining stream water that contains $p\%$ of the current pollution levels is given by

$$C(p) = \frac{285{,}000}{p} - 2{,}850$$

    euros. Because p is the percentage of current levels, $0 \leq p \leq 100$.

(a) Use the restriction on p and determine the range for C.

(b) Graph the function.

(c) Describe what happens to the cost as p takes on values near 0.

(d) The point (1, 282,150) lies on the graph. Explain its meaning.

(e) Explain the meaning of the p-intercept.

18. Suppose the cost of removing $p\%$ of a certain pollution from the exhaust gases at an industrial site is given by

$$C(p) = \frac{8{,}100p}{100 - p}$$

euros. Because p is the pollution, $0 \leq p \leq 100$.

(a) Use the restriction on p and determine the range for C.

(b) Graph the function.

(c) Describe what happens to the cost as p takes on values near 0.

(d) The point (98, 396,900) lies on the graph. Explain its meaning.

(e) Explain the meaning of the p-intercept.