The Graph of a Function

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Aims and Objectives

- The notion of the graph of a function: sketching it and applying to reveal additional information about the function.
- The graph of a quadratic function: sketching a parabola and applying to optimization problems.

Contents

The Graph of a Function

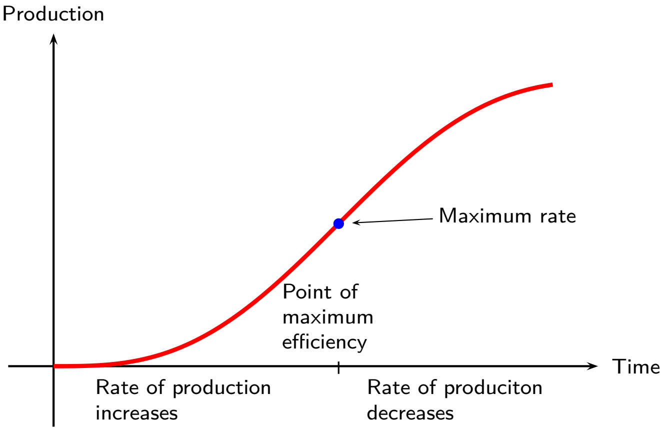

Figure: The output of a factory worker.

The Graph of a Function. (Continued)

The Graph of a Function

The *graph* of a function f consists of all points (x, y),
where x is in the domain of f and $y = f(x)$;
i.e., all points of the form $(x, f(x))$.

Sketching a Graph

Sketching the Graph of a Function by Plotting Points

1. Chose a representative set of numbers x from the domain of f and construct a table of function values $y = f(x)$ for those numbers.
2. Plot the corresponding points (x, y).
3. Connect the plotted points with a smooth curve.

Example of Sketching a Graph

Example

Graph the function $y = x^2$.

Solution.

Begin by constructing the table

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
$y = f(x)$	9	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4	9

Then plot the points (x, y) and connect them with the smooth curve shown in the following figure. $\square$

Shembull skicimi të një grafiku. (Vazhdim)

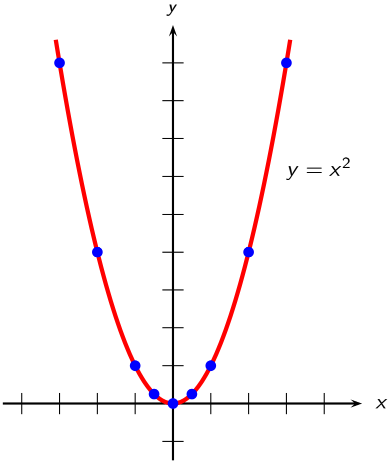

Figure: The graph of $y = x^2$.

x- and y-Intercepts of a Graph

How to Find the x- and y-Intercepts

To find any y-intercept of $y = f(x)$, set $x = 0$ and solve for y.
To find any x-intercept of $y = f(x)$, set $y = 0$ and solve for x.

Example of x- and y-Intercepts

Example

Graph the function $y = -x^2 - x + 2$.
Include all x- and y-intercepts.

Solution...

The y-intercept is $f(0) = 2$.
To find the x-intercepts, solve the equation

$$f(x) = 0;$$

i.e.,

$$-x^2 - x + 2 = 0.$$

□

Example of x- and y-Intercepts. (Continued)

. . . Solution. . .

Recollect the *quadratic formula*, according to which the equation

$$ax^2 + bx + c = 0$$

has real solutions if and only if $D = b^2 - 4ac \geq 0$,
in which case the solutions are

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

□

Example of x- and y-Intercepts. (Continued)

. . . Solution. . .

For our equation, we have $a = -1$, $b = -1$, $c = 2$, therefore

$$x_1 = \frac{-(-1) + \sqrt{(-1)^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} = -2,$$

$$x_2 = \frac{-(-1) - \sqrt{(-1)^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} = 1.$$

Hence, x-intercepts are $(-2, 0)$ and $(1, 0)$. $\square$

Example of x- and y-Intercepts. (Continued)

. . . Solution.

Next, make a table of values
and plot the corresponding points $(x, f(x))$.

x	-4	-3	-2	-1	0	1	2	3
$y = f(x)$	-10	-4	0	2	2	0	-4	-10

The graph of f is shown in the following figure. □

Example of x- and y-Intercepts. (Continued)

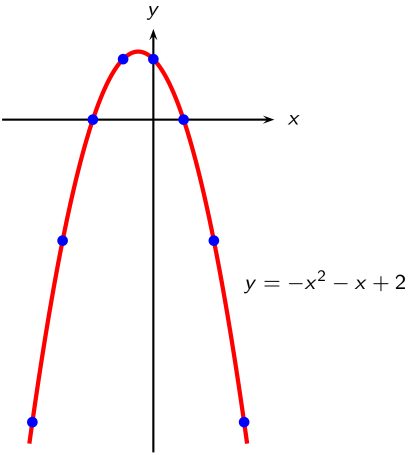

Figure: The graph of $y = -x^2 - x + 2$.

The Graph of a Quadratic Function

Parabola

The graph of a *quadratic function*

$$y = ax^2 + bx + c,$$

for $a \neq 0$, is called a *parabola*.

Exapmle 1. Example 2.

Graphing Parabolas

Remember!

To sketch a parabola $y = ax^2 + bx + c$,
you need only determine three key features:

1. The location of the vertex, where

$$x = -\frac{b}{2a};$$

2. Whether the parabola opens up $(a > 0)$ or down $(a < 0)$;
3. x- and y-intercepts.

Shembulli 1. Shembulli 2.

The Graph of the Parabola $y = ax^2 + bx + c$

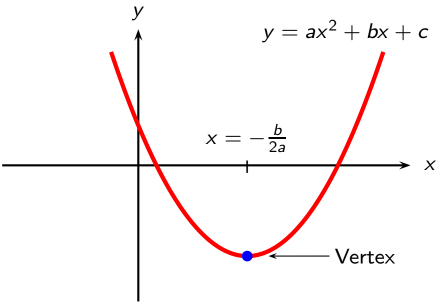

Figure: If $a > 0$, the parabola opens up.

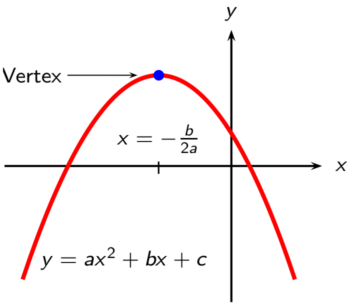

Figure: If $a < 0$, the parabola opens down.

Example of Applying a Parabola

Example

Suppose that $50 - x$ of a certain commodity will be sold when the price is x euros per unit.
What price corresponds to maximum revenue?

Solution.

The revenue derived from selling x units is given by the function

$$R(x) = (50 - x)x = -x^2 + 50x,$$

whose graph is downward opening parabola with vertex at

$$x = -\frac{50}{2 \cdot (-1)} = 25.$$

So, the largest possible revenue is obtained by charging 25 € per unit. $\square$

Example of Applying a Parabola. (Continued)

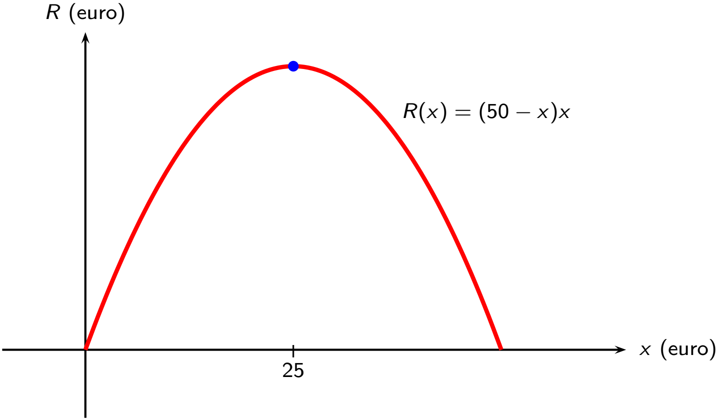

Figure: The graph of the *revenue function*.

For Further Reading

- `http://fberisha.netfirms.com`
- Homework: Exercises from teaching materials
- L. D. Hofmann, G. L. Bradley, *Calculus – for business, economics and life sciences,* pp. 16–29
- F. M. Berisha, M. Q. Berisha, *Matematikë – për biznes dhe ekonomiks,* pp. 113–122

Summary

- Graph of a function: the points $(x, f(x))$
- x- and y-intercepts
- Parabola: sketching and applying to optimization problems
- Polynomial. Rational function.
  (Review from section: Sequences and Sequence Limits.)